



## CHAPTER 04

# Algebra I

## Exponents, radicals and polynomial operations

Classes 21–26 . January 12–28, 2027

Tuesdays and Thursdays . 6:00–7:00 PM Central

### DIFFERENT FORMS, THE SAME VALUE

$$x^3 \times x^2 = x^5$$

Combine powers with a reason.

$$\sqrt{72} = 6\sqrt{2}$$

Keep an exact value in a simpler form.

$$(x + 2)(x + 3)$$

Multiply to get  $x^2 + 5x + 6$ .

Clear explanations . Worked examples . Precise diagrams  
Short practice checks . Worked answers . Chapter revision

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# Your route through Chapter 4

Open your next lesson or revisit a topic. Each lesson includes explanations, worked examples and short practice checks with answers.

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Chapter quick reference: page 36 . Study guidance and sources: page 37

Use these pages during each 60-minute class and for revision afterward. Complete the digital practice your teacher assigns separately.

## LESSON 21 | MEET THE IDEA

# Integer exponent laws

Tuesday, January 12, 2027 . 6:00–7:00 PM Central

### By the end, I can...

Use integer exponent laws, explain zero and negative exponents, and keep the restrictions that make an expression valid.

An **exponent** tells how many copies of the base are multiplied when the exponent is a positive integer. The base and exponent have different jobs.

$$2^3 = 2 \times 2 \times 2 = 8$$

base 2      exponent 3

Three factors of 2, not  $2 \times 3$ .

### Why we add exponents when multiplying the same base

$$x^3 \times x^2$$

Write the factors:  $(x \times x \times x)(x \times x)$ .

$$x^{3+2} = x^5$$

There are five factors of  $x$  altogether.

### Product rule

$$a^m \times a^n = a^{m+n}$$

The bases must match. For example,  $2^3 \times 3^2$  cannot become  $6^5$ . Work out  $8 \times 9 = 72$  instead.

LESSON 21 | THREE MORE RULES

# Divide powers and raise a power

**Quotient rule** |  $x^5 / x^2$ , with  $x \neq 0$

$(x \times x \times x \times x \times x)$   
 $/ (x \times x)$   
 $x^{5-2} = x^3$

Cancel two nonzero factors from top and bottom.

Subtract the denominator exponent from the numerator exponent.

| Rule                               | Example                 |
|------------------------------------|-------------------------|
| $a^m / a^n = a^{m-n}$ , $a \neq 0$ | $a^7 / a^3 = a^4$       |
| $(a^m)^n = a^{mn}$                 | $(x^3)^2 = x^6$         |
| $(ab)^n = a^n b^n$                 | $(2x^3)^2 = 4x^6$       |
| $(a/b)^n = a^n / b^n$ , $b \neq 0$ | $(3x/y)^2 = 9x^2 / y^2$ |

For positive integer powers, these rules apply wherever the original expression is defined. With zero or negative exponents, nonzero-base restrictions also matter; the next page explains them.

**Product or power of a power?**

$x^3 \times x^2 = x^5$ : add  $3 + 2$ .  
 $(x^3)^2 = x^6$ : multiply  $3 \times 2$ .  
Read the operation before choosing a rule.

## LESSON 21 | EXTEND A PATTERN

# Zero and negative exponents

As the exponent drops by 1, divide the value by 2. Keep that pattern going through zero and below.

| Power | $2^3$ | $2^2$ | $2^1$ | $2^0$ | $2^{-1}$ | $2^{-2}$ |
|-------|-------|-------|-------|-------|----------|----------|
| Value | 8     | 4     | 2     | 1     | $1/2$    | $1/4$    |

Why  $a^0 = 1$  when  $a \neq 0$

$$a^3 / a^3 = 1$$

A nonzero quantity divided by itself is 1.

$$a^{3-3} = a^0$$

The quotient rule gives the same value.

A negative exponent means a reciprocal

For  $a \neq 0$  and positive integer  $n$ :

$$a^{-n} = 1/a^n.$$

Examples:  $5^{-2} = 1/25$ ;  $x^{-3} = 1/x^3$ ,  $x \neq 0$ . It does **not** mean  $-a^n$ .

## Keep parentheses around a negative base

$(-2)^4 = 16$ : four negative factors multiply to a positive number.

$-2^4 = -16$ : raise 2 to the fourth power, then apply the minus sign.

Zero raised to a positive integer power is 0. Zero to a negative power would require division by 0, so it is undefined. In this course, we also leave  $0^0$  undefined.

## LESSON 21 | KEEP TRACK OF EVERY FACTOR

# Combine the rules carefully

Simplify  $(6x^5y^{-2}) / (3x^2y)$ , with  $x \neq 0$  and  $y \neq 0$

$$2x^{5-2}y^{-2-1}$$

Divide coefficients; subtract exponents for matching bases.

$$2x^3y^{-3}$$

Simplify the exponent calculations.

$$2x^3 / y^3$$

Rewrite with positive exponents.

Keep  $x \neq 0$  and  $y \neq 0$  from the original expression, even though the final expression no longer has  $x$  in its denominator.

**A powers-of-ten example**

$$(1.2 \times 10^3)(3 \times 10^2)$$

A model has 1,200 groups, each with 300 objects.

$$(1.2 \times 3) \times 10^{3+2}$$

Multiply the coefficients and add the powers of 10.

$$3.6 \times 10^5 = 360,000$$

The model contains 360,000 objects.

**Do not use a multiplication rule for addition**

$$x^2 + x^2 = 2x^2, \text{ not } x^4.$$

$$x^2 \times x^2 = x^4.$$

The operation sign tells you what to do.

## LESSON 21 | SHORT CHECKS

# Your turn

Try these before the worked answers. Write positive exponents in final answers and keep the stated restrictions.

1. Simplify  $x^4 \times x^3$ .

---

2. Simplify  $a^8 / a^3$ , with  $a \neq 0$ .

---

3. Simplify  $(3p^2q^3)^2$ .

---

4. Rewrite  $4x^{-2}$ , with  $x \neq 0$ , using a positive exponent.

---

5. Simplify  $(8a^5b^{-1}) / (2a^2b^2)$ , with  $a \neq 0$  and  $b \neq 0$ .

---

6. Evaluate  $(-3)^2$  and  $-3^2$ . Explain why the signs differ.

---

7. Calculate  $(2 \times 10^3)(4 \times 10^2)$ . Give a power-of-ten form and an ordinary number.

---

## LESSON 21 | WORKED ANSWERS

# Check the rule you chose

1.  $x^{4+3} = x^7$ . Same base, multiplication: add the exponents.
2.  $a^{8-3} = a^5$ , keeping  $a \neq 0$  from the original denominator.
3.  $3^2(p^2)^2(q^3)^2 = 9p^4q^6$ . Square every factor.
4.  $4/x^2$ ,  $x \neq 0$ . Only  $x$  has exponent  $-2$ ; the coefficient  $4$  is not squared or inverted.
5.  $4a^{5-2}b^{-1-2} = 4a^3b^{-3} = 4a^3/b^3$ , with  $a \neq 0$  and  $b \neq 0$ .
6.  $(-3)^2 = 9$ , because  $(-3)(-3) = 9$ .  $-3^2 = -9$ , because the exponent applies to  $3$  before the minus sign.
7.  $(2 \times 4) \times 10^{3+2} = 8 \times 10^5 = 800,000$ .

### A useful self-check

Try a simple allowed value such as  $x = 2$  in your original and simplified expressions. Different values expose an error. One matching test supports your work but does not prove two expressions are always equivalent.

## I can...

- ☐ Distinguish multiplication from a power of a power.
- ☐ Interpret zero and negative exponents.
- ☐ Keep nonzero restrictions from the original expression.

Next class: square roots and fractional exponents. Curriculum focus: A.11B.

LESSON 22 | MEET THE IDEA

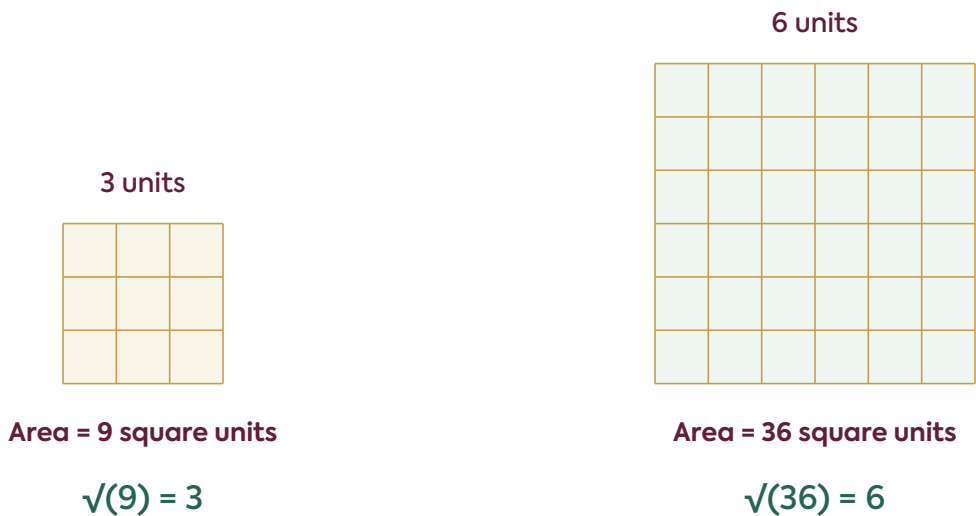
# Square roots and rational exponents

Thursday, January 14, 2027 . 6:00–7:00 PM Central

**By the end, I can...**

Simplify numerical square roots and expressions with rational exponents, using real-number and nonzero restrictions correctly.

Squaring a side length gives a square's area. Taking a **square root** reverses that step. The symbol  $\sqrt{\phantom{x}}$  means the **principal square root**: the nonnegative one.



Each small square is 1 square unit.

**A square root and an equation are different**

$\sqrt{(9)} = 3$ : the radical symbol names one nonnegative value.  
 $x^2 = 9$ : the equation has two solutions,  $x = 3$  and  $x = -3$ .  
 Both numbers square to 9, but  $\sqrt{(9)}$  does not mean  $\pm 3$ .

We write parentheses after  $\sqrt{\phantom{x}}$  to make clear which expression is inside the root. Over the real numbers, a square root needs a nonnegative radicand:  $\sqrt{(0)} = 0$ , while  $\sqrt{(-9)}$  is not a real number.

## LESSON 22 | EXACT RADICAL FORM

# Look for a perfect-square factor

**Simplify  $\sqrt{72}$**

$$\sqrt{72} = \sqrt{36 \times 2}$$

Choose the largest perfect-square factor: 36.

$$\sqrt{36} \times \sqrt{2}$$

A square root of a product can split this way when both factors are nonnegative.

$$6\sqrt{2}$$

Since  $\sqrt{36} = 6$ , take 6 outside the root.

$6\sqrt{2}$  is exact. Its decimal approximation is about 8.49; use  $\approx$  for a rounded value.

**Combine roots only after simplifying**

$$2\sqrt{8} + \sqrt{18}$$

Start with a sum of radicals.

$$2(2\sqrt{2}) + 3\sqrt{2}$$

Use  $8 = 4 \times 2$  and  $18 = 9 \times 2$ .

$$4\sqrt{2} + 3\sqrt{2} = 7\sqrt{2}$$

The radical parts match, so add their coefficients.

**Multiplication and division of numerical roots**

$$\sqrt{12} \times \sqrt{3} = \sqrt{36} = 6.$$

$$\sqrt{48} / \sqrt{3} = \sqrt{16} = 4.$$

These real-root product rules require nonnegative radicands; a radicand in a denominator must be positive.

LESSON 22 | READ THE NUMERATOR AND DENOMINATOR

# A fractional exponent names a root

A **rational exponent** is an exponent written as a fraction. For a **positive base a**, with positive integer n,  $a^{1/n}$  means the nth root of a.

| Expression | Read it as                          | Example              |
|------------|-------------------------------------|----------------------|
| $a^{1/2}$  | Square root of a                    | $25^{1/2} = 5$       |
| $a^{1/3}$  | Cube root of a                      | $27^{1/3} = 3$       |
| $a^{m/n}$  | Take the nth root, then raise to m. | $27^{2/3} = 3^2 = 9$ |

Example 1 |  $16^{3/4}$

$16^{1/4} = 2$

$(16^{1/4})^3 = 2^3$

$16^{3/4} = 8$

The denominator 4 names a fourth root:  $2^4 = 16$ .

The numerator 3 tells you the power.

Root first keeps the numbers manageable.

Example 2 |  $16^{-1/2}$

$1 / 16^{1/2}$

$1 / 4$

The negative sign in the exponent means a reciprocal.

Take the square root of 16.

LESSON 22 | ALGEBRAIC POWERS AND RESTRICTIONS

# The same laws, with safe bases

For the rational-exponent laws on this page, assume  $x > 0$  and  $a > 0$ . This keeps all the roots and quotients real and defined.

**Multiply, divide and raise a power**

|                                |                                  |
|--------------------------------|----------------------------------|
| $x^{1/2} \times x^{3/2} = x^2$ | Add exponents: $1/2 + 3/2 = 2$ . |
| $a^{5/3} / a^{2/3} = a$        | Subtract: $5/3 - 2/3 = 1$ .      |
| $(x^{3/2})^2 = x^3$            | Multiply: $(3/2) \times 2 = 3$ . |

| Question          | What matters                                                                       |
|-------------------|------------------------------------------------------------------------------------|
| $\sqrt{((-5)^2)}$ | First square $-5$ to get $25$ ; its principal square root is <b>5</b> , not $-5$ . |
| $\sqrt{(-16)}$    | No real value: a real square is never negative.                                    |
| Cube root of $-8$ | It is $-2$ , because $(-2)^3 = -8$ . Odd roots can be negative.                    |
| $0^{-1/2}$        | Undefined: it would require $1/\sqrt{0}$ , which divides by zero.                  |

**A root does not distribute over a sum**

$\sqrt{9 + 16} = \sqrt{25} = 5$ .  
 But  $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$ .  
 The product rule for roots is not a rule for addition.

## LESSON 22 | SHORT CHECKS

# Your turn

Give exact radical forms unless a decimal is requested. In Question 6, assume  $x > 0$ .

1. Simplify  $\sqrt{50}$ . Then approximate it to two decimal places.

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2. Simplify  $3\sqrt{12} - \sqrt{27}$ .

---

3. Simplify  $\sqrt{20} \times \sqrt{5}$ .

---

4. Evaluate  $81^{3/4}$  and  $25^{-1/2}$ .

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5. Find  $\sqrt{49}$ , then solve  $t^2 = 49$ . Explain the difference.

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6. Simplify  $(x^{5/2} \times x^{1/2}) / x$ . Keep the stated restriction.

---

7. Is  $\sqrt{16 + 9}$  equal to  $\sqrt{16} + \sqrt{9}$ ? Check both sides.

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## LESSON 22 | WORKED ANSWERS

# Check your roots and powers

1.  $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$ , which is approximately **7.07**. The radical form is exact; the decimal is rounded.
2.  $3\sqrt{12} - \sqrt{27} = 3(2\sqrt{3}) - 3\sqrt{3} = 6\sqrt{3} - 3\sqrt{3} = 3\sqrt{3}$ .
3.  $\sqrt{20} \times \sqrt{5} = \sqrt{100} = 10$ . Both radicands are nonnegative.
4.  $81^{3/4} = (81^{1/4})^3 = 3^3 = 27$ . Also,  $25^{-1/2} = 1/\sqrt{25} = 1/5$ .
5.  $\sqrt{49} = 7$ . The equation  $t^2 = 49$  has  **$t = 7$  or  $t = -7$** . The symbol  $\sqrt{\phantom{x}}$  selects the principal, nonnegative root; the equation asks for every value whose square is 49.
6.  $x^{5/2+1/2-1} = x^{3-1} = x^2$ , with  $x > 0$  as specified.
7. **No.**  $\sqrt{25} = 5$ , while  $\sqrt{16} + \sqrt{9} = 4 + 3 = 7$ .

### Exact or approximate?

Use  $=$  between exactly equal forms, such as  $\sqrt{50} = 5\sqrt{2}$ . Use  $\approx$  for rounded forms, such as  $\sqrt{50} \approx 7.07$ .

## I can...

- ☐ Extract perfect-square factors and combine matching radicals.
- ☐ Read fractional exponents and use the exponent laws.
- ☐ Distinguish a principal root from the solutions of an equation.

Next class: organize, add and subtract polynomials. Curriculum focus: A.11A, A.11B.

LESSON 23 | MEET THE IDEA

# Adding, subtracting and organizing polynomials

Tuesday, January 19, 2027 . 6:00–7:00 PM Central

**By the end, I can...**

Organize polynomial terms by degree, combine like terms and rewrite linear and quadratic expressions without changing their value.

A **polynomial in  $x$**  is a sum of terms such as  $4x^2$ ,  $-3x$  and  $7$ . Its variable exponents are nonnegative integers. A term’s **coefficient** is its numerical factor.

| Polynomial      | Degree       | Number of terms |
|-----------------|--------------|-----------------|
| $4x^2 - 3x + 7$ | 2: quadratic | 3: trinomial    |
| $5x - 8$        | 1: linear    | 2: binomial     |
| $-2x^2$         | 2: quadratic | 1: monomial     |
| $6$             | 0: constant  | 1: monomial     |

**Degree is not the number of terms**

For a nonzero polynomial in one variable, the degree is the highest exponent with a nonzero coefficient. The expression  $7x^2 - 9$  has degree 2 and two terms;  $x^2$  has degree 2 and only one term.

Write polynomials in **descending powers**: for example,  $5 - 2x + 3x^2$  becomes  $3x^2 - 2x + 5$ . The coefficient of  $x$  is  $-2$ , including its sign.

Nonexamples:  $1/x = x^{-1}$  and  $\sqrt{x} = x^{1/2}$  are not polynomials in  $x$ . Their exponents are not nonnegative integers.

## LESSON 23 | ADDITION AND SUBTRACTION

# Match like terms before combining

**Like terms** have exactly the same variable part, including exponents. You may combine  $3x^2$  and  $x^2$ , but not  $3x^2$  and  $x$ .

**Add** |  $(3x^2 - 2x + 5) + (x^2 + 7x - 4)$

$$\begin{aligned} &(3x^2 + x^2) \\ &+ (-2x + 7x) + (5 - 4) \\ &4x^2 + 5x + 1 \end{aligned}$$

Group the quadratic terms, linear terms and constants.

Add coefficients; keep each variable part unchanged.

**Subtract** |  $(5x^2 + x - 3) - (2x^2 - 4x + 7)$

$$\begin{aligned} &5x^2 + x - 3 \\ &- 2x^2 + 4x - 7 \\ &(5 - 2)x^2 \\ &+ (1 + 4)x + (-3 - 7) \\ &3x^2 + 5x - 10 \end{aligned}$$

Distribute -1 to every term in the second parentheses.

Group like terms after changing the signs.

Combine coefficients and constants.

**Keep the exponent**

$$3x^2 + x^2 = 4x^2, \text{ not } 4x^4.$$

Adding like terms changes their coefficient, not their exponent.

LESSON 23 | KEEP THE STRUCTURE VISIBLE

# Missing terms and equivalent forms

Use a zero coefficient to align a missing term. It adds nothing, but helps you keep the columns straight.

| Operation | $x^2$ terms | $x$ terms | Constants |
|-----------|-------------|-----------|-----------|
| Start     | $4x^2$      | $0x$      | 6         |
| Subtract  | $x^2$       | $-2x$     | 1         |
| Result    | $3x^2$      | $2x$      | 5         |

So  $(4x^2 + 6) - (x^2 - 2x + 1) = 3x^2 + 2x + 5$ . In the middle column,  $0x - (-2x) = 2x$ .

Distribution works in both directions

$3x(2x + 3) = 6x^2 + 9x$

Expand by multiplying 3x by both terms.

$6x^2 + 9x = 3x(2x + 3)$

Read backward to write a common-factor form.

These are equivalent expressions. The next chapter develops factoring methods in more detail.

## A perimeter expression

A rectangle has length  $2x + 3$  and width  $x + 1$ , measured in centimeters, with  $x > 0$ .

$$P = 2(2x + 3) + 2(x + 1) = 4x + 6 + 2x + 2 = \mathbf{6x + 8 \text{ cm.}}$$

At  $x = 2$ , its sides are 7 cm and 3 cm, and its perimeter is 20 cm.

## LESSON 23 | SHORT CHECKS

# Your turn

Write answers in descending powers. Show the changed signs before combining a subtraction.

1. Add  $(2x^2 + 3x - 4) + (x^2 - 5x + 9)$ .

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2. Subtract  $(4x^2 - x + 2) - (x^2 + 2x - 6)$ .

---

---

3. Simplify  $(3x^2 + 5) - (x^2 - 2x + 1)$ . Include the missing  $x$  term in your work.

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4. Rewrite  $8x^2 + 4x$  as  $4x$  multiplied by a bracketed expression. Expand to check.

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5. A rectangle has sides  $3x + 2$  and  $x + 4$  centimeters, with  $x > 0$ . Write and simplify its perimeter.

---

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6. For  $7x^2 - 9$ , name the degree, leading coefficient and number of terms.

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## LESSON 23 | WORKED ANSWERS

# Check the coefficients and signs

1.  $(2 + 1)x^2 + (3 - 5)x + (-4 + 9) = 3x^2 - 2x + 5$ .

2.  $4x^2 - x + 2 - x^2 - 2x + 6 = 3x^2 - 3x + 8$ . The  $-6$  inside the subtracted polynomial becomes  $+6$ .

3.  $3x^2 + 0x + 5 - x^2 + 2x - 1 = 2x^2 + 2x + 4$ .

4.  $4x(2x + 1)$ . Expanding gives  $4x \times 2x + 4x \times 1 = 8x^2 + 4x$ .

5.  $P = 2(3x + 2) + 2(x + 4) = 6x + 4 + 2x + 8 = 8x + 12$  cm. Keep the contextual restriction  $x > 0$ .

6. **Degree 2; leading coefficient 7; two terms**, so it is a quadratic binomial. The constant term is  $-9$ .

**Explain the most common subtraction error**

Subtracting a polynomial means subtracting **all** its terms. Write a second line with no parentheses and the signs changed before you combine anything.

## I can...

- ☐ Identify degree and group like terms.
- ☐ Distribute a minus sign across a whole polynomial.
- ☐ Connect expanded and common-factor forms.

Next class: multiply polynomials systematically. Curriculum focus: A.10A, A.10D.

LESSON 24 | MEET THE IDEA

# Multiplying polynomials

Thursday, January 21, 2027 . 6:00–7:00 PM Central

By the end, I can...

Multiply every term of one polynomial by every term of the other, combine like terms and check the result.

Multiplication uses the distributive property. For  $(x + 3)(x + 2)$ , each term in the first factor must meet **both** terms in the second factor.

Build  $(x + 3)(x + 2)$

|   |       |      |
|---|-------|------|
|   | x     | 2    |
| x | $x^2$ | $2x$ |
| 3 | $3x$  | 6    |

This is an algebra grid, not a scale drawing.

Add the four cells:  
 $x^2 + 2x + 3x + 6 = x^2 + 5x + 6.$

Count the products

Two terms multiplied by two terms give **four partial products** before combining. The grid helps you avoid missing one. With three terms in a factor, keep the same every-term-by-every-term rule.

## LESSON 24 | DISTRIBUTE, THEN COMBINE

# Signs travel with their terms

**Multiply  $(2x - 3)(x + 4)$**

$$2x(x + 4) - 3(x + 4)$$

$$2x^2 + 8x - 3x - 12$$

$$2x^2 + 5x - 12$$

Use the complete signed terms  $2x$  and  $-3$ .

Distribute each term to the whole second factor.

Combine the linear terms.

## Check with a simple input

At  $x = 2$ , the original product is  $(4 - 3)(2 + 4) = 1 \times 6 = 6$ .

The expanded expression gives  $2(4) + 5(2) - 12 = 6$ .

The values agree. This catches many errors, though one matching input is not a proof of an identity.

**A single term also distributes**

$$2x(3x + 4)$$

$$6x^2 + 8x$$

Multiply  $2x$  by both terms.

Use  $x \times x = x^2$ ; coefficients multiply too.

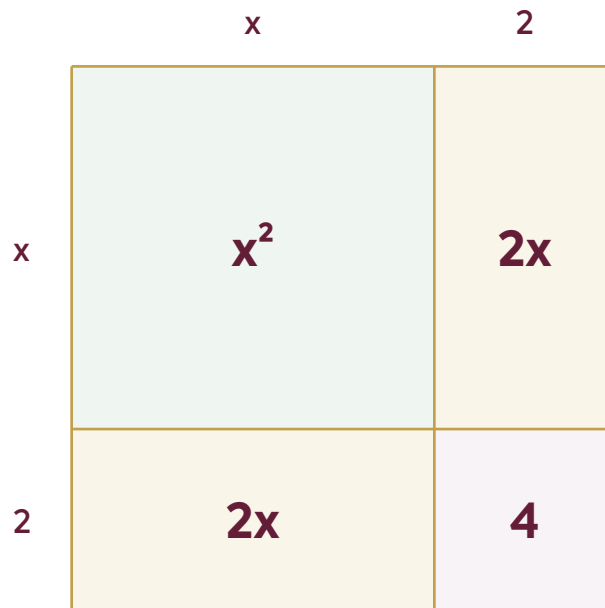
**The sign belongs to the whole term**

In the first example,  $-3 \times 4 = -12$ . Do not drop the minus sign or change it just because you are combining other terms.

LESSON 24 | AN AREA MODEL

# Why a squared binomial has a middle term

$(x + 2)^2$  means  $(x + 2)(x + 2)$ . A square with that side length splits into four regions, not two.



$$(x + 2)^2 = x^2 + 2x + 2x + 4$$

Area model for positive  $x$ ; illustration uses  $x = 4$ .

Both rectangles matter:  $2x + 2x = 4x$ . So

$$(x + 2)^2 = x^2 + 4x + 4.$$

## The general pattern

$$(a + b)^2 = a^2 + 2ab + b^2.$$

$$(a - b)^2 = a^2 - 2ab + b^2.$$

The identities work algebraically for all real  $a$  and  $b$ . The area picture uses positive lengths.

For example,  $(x - 3)^2 = (x - 3)(x - 3) = x^2 - 6x + 9$ . The last term is positive because  $(-3)(-3) = 9$ .

LESSON 24 | LINEAR AND QUADRATIC FACTORS

# The product can have a higher degree

Linear × quadratic |  $(x + 2)(x^2 - x + 3)$

$$x(x^2 - x + 3)$$

$$+ 2(x^2 - x + 3)$$

$$x^3 - x^2 + 3x$$

$$+ 2x^2 - 2x + 6$$

$$x^3 + x^2 + x + 6$$

Distribute each term across all three terms.

There are six partial products.

Combine like terms. A degree-3 result is expected.

Quadratic × quadratic:  $(x^2 + 1)(x^2 + x + 2)$

|       |       |       |        |
|-------|-------|-------|--------|
|       | $x^2$ | $x$   | $2$    |
| $x^2$ | $x^4$ | $x^3$ | $2x^2$ |
| $1$   | $x^2$ | $x$   | $2$    |

This is an algebra grid, not a scale drawing.

Add the cells:  $x^4 + x^3 + 3x^2 + x + 2$ .

The factors each have degree 2; their product has degree 4. A factor does not need every possible term to be quadratic.

Today’s input factors have degree 1 or 2, but their products are not restricted to degree 2. The same distributive process handles all these examples.

## LESSON 24 | SHORT CHECKS

# Your turn

Use a grid or distribution. Show the partial products before combining.

1. Expand  $(x + 4)(x - 1)$ .

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2. Expand  $(2x - 1)(x + 3)$ .

---

---

3. Expand  $(x + 1)(x^2 + 2x + 3)$ .

---

---

4. Expand  $(x^2 + 2)(x^2 - 1)$ .

---

---

5. A square has side  $x + 5$  centimeters, with  $x > 0$ . Write its area as a polynomial. Explain the middle term.

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6. A student claims  $(x + 3)^2 = x^2 + 9$ . Test  $x = 1$ , then correct the expansion.

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## LESSON 24 | WORKED ANSWERS

# Check every partial product

1.  $x^2 - x + 4x - 4 = x^2 + 3x - 4.$

2.  $2x^2 + 6x - x - 3 = 2x^2 + 5x - 3.$

3.  $x^3 + 2x^2 + 3x + x^2 + 2x + 3 = x^3 + 3x^2 + 5x + 3.$

4.  $x^4 - x^2 + 2x^2 - 2 = x^4 + x^2 - 2.$

5.  $A = (x + 5)^2 = x^2 + 10x + 25$  **square centimeters**. Two rectangles each contribute  $5x$ , so the middle term is  $10x$ . Keep  $x > 0$  for the stated length model.

6. At  $x = 1$ , the actual square is  $4^2 = 16$ , but the claimed result is  $1 + 9 = 10$ . The correct expansion is  $x^2 + 6x + 9$ . The missing products are  $3x$  and  $3x$ .

**A quick product audit**

Did every term meet every term? Did each sign stay attached? Did you add only like terms? A final substitution can expose an arithmetic slip.

**I can...**

- ☐ Multiply linear and quadratic factors systematically.
- ☐ Explain the middle term in a squared binomial.
- ☐ Check a result without treating one test value as proof.

Next class: reverse multiplication with polynomial division. Curriculum focus: A.10B, A.10D.

LESSON 25 | MEET THE IDEA

# Dividing linear and quadratic polynomials

Tuesday, January 26, 2027 . 6:00–7:00 PM Central

**By the end, I can...**  
Divide linear and quadratic polynomials, express any remainder and keep the original divisor nonzero.

Division asks: **what multiplies the divisor to make the dividend?** If it does not divide exactly, keep the leftover part as a remainder.

| Word      | Meaning                             | In $17 \div 5$ |
|-----------|-------------------------------------|----------------|
| Dividend  | What you divide                     | 17             |
| Divisor   | What you divide by                  | 5              |
| Quotient  | The whole-number part here          | 3              |
| Remainder | What is left after multiplying back | 2              |

$17 = 5 \times 3 + 2$ , so  $17/5 = 3 + 2/5$ .  
For polynomials, the same structure is **dividend = divisor  $\times$  quotient + remainder**.

**Divide by a monomial |  $(6x^2 + 9x) / (3x)$ ,  $x \neq 0$**

$6x^2/(3x) + 9x/(3x)$   
 $2x + 3$   
Keep  $x \neq 0$ , because the original divisor is  $3x$ .

Divide each numerator term by the same divisor.  
Check:  $3x(2x + 3) = 6x^2 + 9x$ .

## Long division, one step at a time

Divide  $x^2 + 5x + 6$  by  $x + 2$ . The original divisor requires  $x \neq -2$ . Line up matching powers before you subtract.

|         |                                                                                                                                                                    |       |                          |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|--------------------------|
|         | $x$                                                                                                                                                                | $+ 3$ | Quotient: $x + 3$        |
| $x + 2$ | $\begin{array}{r} \overline{) \begin{array}{rrr} x^2 & + 5x & + 6 \\ - x^2 & + 2x & \\ \hline & 3x & + 6 \\ & - 3x & + 6 \\ \hline & & 0 \end{array}} \end{array}$ |       | Dividend: $x^2 + 5x + 6$ |
|         |                                                                                                                                                                    |       | Subtract $x(x + 2)$ .    |
|         |                                                                                                                                                                    |       | Bring down $+6$ .        |
|         |                                                                                                                                                                    |       | Subtract $3(x + 2)$ .    |
|         |                                                                                                                                                                    |       | Remainder: 0             |

| Step                           | Reason                                                    |
|--------------------------------|-----------------------------------------------------------|
| $x^2 \div x = x$               | Choose $x$ as the first term of the quotient.             |
| Subtract $x(x + 2) = x^2 + 2x$ | This removes the leading $x^2$ term and leaves $3x + 6$ . |
| $3x \div x = 3$                | Choose $+3$ as the next quotient term.                    |
| Subtract $3(x + 2) = 3x + 6$   | The remainder is 0, so the division is exact.             |

**Result:**  $x + 3$ , with  $x \neq -2$ . Multiplying  $(x + 2)(x + 3)$  gives the original  $x^2 + 5x + 6$ . The restriction remains even when the quotient has no denominator.

## LESSON 25 | WHEN DIVISION IS NOT EXACT

# A remainder still belongs in the answer

Divide  $x^2 + 3x + 5$  by  $x + 1$ , with  $x \neq -1$

$$x^2 \div x = x$$

The first quotient term is  $x$ .

$$\begin{array}{r} (x^2 + 3x + 5) \\ - (x^2 + x) \\ \hline 2x + 5 \end{array}$$

Multiply  $x(x + 1)$ , then subtract the whole product.

$$2x \div x = 2$$

The next quotient term is  $+2$ .

$$(2x + 5) - (2x + 2) = 3$$

Subtract  $2(x + 1)$ . The remainder is  $3$ .

Write quotient plus remainder divided by divisor

$$(x^2 + 3x + 5)/(x + 1) = x + 2 + 3/(x + 1), x \neq -1.$$

The remainder is not just an extra  $+3$ ; it is divided by the original divisor.

## Check by rebuilding the dividend

$$\begin{aligned} &(x + 1)(x + 2) + 3 \\ &= x^2 + 3x + 2 + 3 \\ &= x^2 + 3x + 5. \text{ The original dividend is recovered.} \end{aligned}$$

Stop when the remainder is zero or its degree is less than the divisor's degree. Here the remainder has degree 0 and the divisor has degree 1.

## LESSON 25 | CONSTANT QUOTIENTS AND REMINDERS

# When the degrees are equal

Linear  $\div$  linear |  $(5x + 7)/(x + 1)$ ,  $x \neq -1$

$$5x \div x = 5$$

The quotient is the constant 5.

$$(5x + 7) - 5(x + 1) = 2$$

The remainder is 2.

$$5 + 2/(x + 1)$$

Write the remainder over the original divisor.

Quadratic  $\div$  quadratic |  $(3x^2 + 2x + 5)/(x^2 + 1)$

$$3x^2 \div x^2 = 3$$

The quotient is the constant 3.

$$\begin{array}{r} (3x^2 + 2x + 5) \\ - 3(x^2 + 1) \\ \hline \end{array} = 2x + 2$$

Subtract the entire product. The remainder has degree 1.

$$3 + (2x + 2)/(x^2 + 1)$$

The remainder degree is lower than the divisor degree, so stop.

For real  $x$ ,  $x^2 + 1$  is always positive. There are no excluded real  $x$ -values for this divisor.

## Three reminders

**Missing term:** write  $x^2 + 0x - 3$  to keep columns aligned.

**Remainder:** a polynomial remainder may be negative.

**Cancellation:** cancel common factors, not separate terms in sums. For example,  $(x + 2)/x = 1 + 2/x$ , not 2, with  $x \neq 0$ .

## LESSON 25 | SHORT CHECKS

# Your turn

Give a quotient plus remainder/divisor when needed. State which real  $x$ -values, if any, the original divisor excludes.

1. Divide  $(4x + 9)$  by  $(x + 2)$ .

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2. Divide  $(x^2 + 7x + 12)$  by  $(x + 3)$ .

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3. Divide  $(x^2 - 3)$  by  $(x + 1)$ . Write the missing  $0x$  term first.

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4. Divide  $(2x^2 + x + 4)$  by  $(x^2 + 2)$ .

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5. Simplify  $(6x^2 - 3x)/(3x)$ . Verify your answer by multiplication and retain the restriction.

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## LESSON 25 | WORKED ANSWERS

# Multiply back to check

1. Quotient 4, remainder 1:  $4 + 1/(x + 2)$ ,  $x \neq -2$ . Check:  $4(x + 2) + 1 = 4x + 9$ .
2. Subtract  $x(x + 3)$  to leave  $4x + 12$ ; subtract  $4(x + 3)$  to leave 0. Result:  $x + 4$ ,  $x \neq -3$ . Check:  $(x + 3)(x + 4) = x^2 + 7x + 12$ .
3. Start with  $x^2 + 0x - 3$ . Subtract  $x(x + 1)$  to leave  $-x - 3$ . The next quotient term is  $-1$ ; subtract  $-1(x + 1)$  to leave  $-2$ . Result:  $x - 1 - 2/(x + 1)$ ,  $x \neq -1$ . Check:  $(x + 1)(x - 1) - 2 = x^2 - 3$ .
4. The quotient is 2; subtract  $2(x^2 + 2)$  to leave  $x$ . Result:  $2 + x/(x^2 + 2)$ . There are **no excluded real values**, because  $x^2 + 2 \geq 2$ . Check:  $2(x^2 + 2) + x = 2x^2 + x + 4$ .
5.  $2x - 1$ , with  $x \neq 0$ . Check:  $3x(2x - 1) = 6x^2 - 3x$ . The original fraction is undefined at  $x = 0$ , even though  $2x - 1$  is defined there.

### The final check has two parts

**Identity:** divisor  $\times$  quotient + remainder = dividend.

**Domain:** the original divisor must not be zero. A correct expansion does not remove an excluded value.

## I can...

- ☐ Divide, multiply, subtract and repeat in aligned columns.
- ☐ Interpret a remainder and know when to stop.
- ☐ Retain the original divisor restrictions.

Next class: review and the Chapter 4 checkpoint. Curriculum focus: A.10C.

LESSON 26 | MEET THE IDEA

# Unit 4 checkpoint and symbolic-fluency repair

Thursday, January 28, 2027 . 6:00–7:00 PM Central

**By the end, I can...**

Choose the right symbolic operation, explain why a step works and repair a specific mistake using evidence from your work.

This is a **revision and correction guide**, not the graded checkpoint. Your teacher supplies the assessment separately. Today introduces no new topic.

| Operation                         | First question to ask                                                         |
|-----------------------------------|-------------------------------------------------------------------------------|
| Exponent rules                    | Am I multiplying, dividing or raising a power? Do the bases match?            |
| Square roots                      | Is there a perfect-square factor? Is the real root defined?                   |
| Polynomial addition / subtraction | Are the terms alike? Have I distributed the minus sign?                       |
| Polynomial multiplication         | Has every term multiplied every term?                                         |
| Polynomial division               | Have I aligned powers, subtracted the whole product and kept the restriction? |

**Our one-hour checkpoint lesson**

- 5 minutes: recall key ideas.
- 20 minutes: a mixed chapter checkpoint.
- 25 minutes: corrections and targeted teaching.
- 10 minutes: exit check and next steps.

LESSON 26 | READ THE OPERATION CAREFULLY

An error can show you what to repair

| Incorrect claim                             | Correction                             | Why                                                         |
|---------------------------------------------|----------------------------------------|-------------------------------------------------------------|
| $x^2 \times x^3 = x^6$                      | $x^5$                                  | Add exponents for multiplication of the same base.          |
| $x^{-2} = -x^2$                             | $1/x^2, x \neq 0$                      | A negative exponent means reciprocal, not a negative value. |
| $\sqrt{(9 + 16)} = 7$                       | $\sqrt{(25)} = 5$                      | A root does not distribute over a sum.                      |
| $(x + 3)^2 = x^2 + 9$                       | $x^2 + 6x + 9$                         | The two cross-products contribute $6x$ .                    |
| $(x^2 + 1) - (x - 2)$<br>$= x^2 - x - 1$    | $x^2 - x + 3$                          | Subtracting $-2$ adds $2$ .                                 |
| Remainder 2 means add $+2$ to the quotient. | Add 2 divided by the original divisor. | The remainder still needs to be divided.                    |

Verify a division result

For  $(x^2 + 3x + 5)/(x + 1)$ : quotient  $x + 2$ , remainder 3.  
 $(x + 1)(x + 2) + 3 = x^2 + 3x + 5$ .  
The result is  $x + 2 + 3/(x + 1)$ , with  $x \neq -1$ .

When a step is wrong, name the rule that should replace it. Then retry a different question that uses the same rule.

LESSON 26 | PRACTICE, NOT THE GRADED CHECKPOINT

# A short revision check

1. Simplify  $(2a^3)^2/a^4$ , with  $a \neq 0$ .
- 
2. Simplify  $\sqrt{(75)}$ , then evaluate  $8^{2/3}$ .
- 
3. Simplify  $(3x^2 - x + 2) - (x^2 + 4x - 5)$ .
- 
4. Expand  $(x - 2)(x + 5)$ .
- 
5. Divide  $(x^2 + 4x + 6)$  by  $(x + 2)$ . State the restriction and check by multiplication.
- 

## My correction log

| Error type                                          | My next action |
|-----------------------------------------------------|----------------|
| Exponent / root / sign / missing product / division |                |
| The rule I need to use                              |                |
| The lesson and fresh question I will revisit        |                |

## LESSON 26 | WORKED ANSWERS AND NEXT STEPS

# Check, correct and move forward

1.  $(2a^3)^2/a^4 = 4a^6/a^4 = 4a^2$ , keeping  $a \neq 0$ .

2.  $\sqrt{(75)} = \sqrt{(25 \times 3)} = 5\sqrt{(3)}$ . Also,  $8^{2/3} = (8^{1/3})^2 = 2^2 = 4$ .

3.  $3x^2 - x + 2 - x^2 - 4x + 5 = 2x^2 - 5x + 7$ .

4.  $x^2 + 5x - 2x - 10 = x^2 + 3x - 10$ .

5. Subtract  $x(x + 2)$  to leave  $2x + 6$ ; subtract  $2(x + 2)$  to leave 2. Result:  $x + 2 + 2/(x + 2)$ , with  $x \neq -2$ . Check:  $(x + 2)(x + 2) + 2 = x^2 + 4x + 6$ .

### Choose one repair, then test it

Revisit Class 21 for exponent laws, Class 22 for roots, Class 23 for signs and like terms, Class 24 for products, or Class 25 for division. Cover the worked answer and try a fresh question. Explain the rule as you use it.

## Ready for the next chapter

- ☐ I can distinguish a sum, product, power and quotient.
- ☐ I can explain a correction, not just copy it.
- ☐ I can retain restrictions and verify an equivalent form.

Chapter 5 starts Tuesday, February 2, 2027: factoring and quadratic functions. Curriculum focus: A.10A-D, A.11A-B.

KEEP THIS PAGE FOR REVISION

# Chapter 4 quick reference

| Idea               | Remember                                                                                    | Example                                                      |
|--------------------|---------------------------------------------------------------------------------------------|--------------------------------------------------------------|
| Product / quotient | For matching bases, add / subtract exponents. A divisor must be nonzero.                    | $x^3x^2 = x^5$ ; $x^5/x^2 = x^3$ , $x \neq 0$ .              |
| Power of a power   | Multiply exponents. Apply an outside power to every factor.                                 | $(2x^3)^2 = 4x^6$ .                                          |
| Zero / negative    | $a^0 = 1$ and $a^{-n} = 1/a^n$ , with $a \neq 0$ .                                          | $2^0 = 1$ ; $2^{-3} = 1/8$ .                                 |
| Numerical roots    | Extract perfect-square factors; $\sqrt{\phantom{x}}$ names the principal nonnegative root.  | $\sqrt{(50)} = 5\sqrt{(2)}$ ; $\sqrt{(9)} = 3$ .             |
| Rational powers    | For positive $a$ , $a^{m/n}$ means $n$ th root, then power $m$ ; $n$ is a positive integer. | $16^{3/4} = 2^3 = 8$ .                                       |
| Add / subtract     | Combine like terms. Distribute a minus sign to every subtracted term.                       | $3x^2 + x^2 = 4x^2$ , not $4x^4$ .                           |
| Multiply           | Every term meets every term; then combine.                                                  | $(x + 2)^2 = x^2 + 4x + 4$ .                                 |
| Divide             | Divisor $\times$ quotient + remainder = dividend.<br>Keep divisor $\neq 0$ .                | $(x^2 + 3x + 5)/(x + 1) = x + 2 + 3/(x + 1)$ , $x \neq -1$ . |

For exponent laws, keep every restriction needed by the original expression. In polynomial division, stop when the remainder is zero or has lower degree than the divisor.

## STUDY GUIDANCE AND CURRICULUM REFERENCES

# Make these notes work for you

## A small routine for each lesson

**Before:** recall the earlier operation named in the lesson.

**During:** predict the next worked step and explain its rule.

**After:** attempt the short checks before reading the answers, then complete the assigned digital practice.

### Practice and assessment have different jobs

These PDFs include answers to their own study checks. The Class 24 quiz and Class 26 graded checkpoint are separate teacher-provided materials. Their unpublished questions and answer keys are not included.

## Precision in symbols and diagrams

Parentheses after  $\sqrt{\phantom{x}}$  identify the complete radicand. Use  $=$  for exact equality and  $\approx$  for rounded values. Area diagrams use positive lengths; algebra grids organize products and are not scale drawings. Every numerical story is a fictional teaching example.

## Curriculum alignment

Standard Algebra I, §111.39(c): A.10A–D and A.11A–B. Advanced Grade 8 Algebra I, §111.31(d): (10)(A)–(D) and (11)(A)–(B). Mathematical processes and earlier expression skills are revisited throughout.

Polynomial multiplication uses linear and quadratic factors, so some products have degree 3 or 4. Division keeps the divisor degree no greater than the dividend degree, with both at most 2. These are original Cambridge Veritas supplementary notes, not an official TEA publication. The dates are our agreed class calendar, not state-required pacing.

## Official references

Texas Education Agency: [Current mathematics standards index](#).

Standard Algebra I: [§111.39, high-school mathematics TEKS](#).

Advanced Grade 8 Algebra I: [§111.31, middle-school mathematics TEKS, adopted 2025](#).

Standards checked September 9, 2026. Chapter 4 edition: 1.0.