



CHAPTER 06

Algebra I

Quadratic equations and modeling

Classes 33–38 . February 23 – March 11, 2027
Tuesdays and Thursdays . 6:00–7:00 PM Central

CHOOSE A METHOD. CHECK THE MEANING.

$$(x - 2)(x - 3) = 0$$

A zero product reveals the roots.

$$(x + 3)^2 = 16$$

A completed square makes the next step clear.

$$h(t) = -t^2 + 3t + 1$$

A fitted model supports a careful prediction.

Four solving methods . Clear worked steps . Accurate graphs
Technology practice . Worked answers . Chapter revision

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Your route through Chapter 6

Learn one method at a time, then compare when each is useful. The short study checks include worked answers at the end of each lesson.

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Chapter quick reference: page 36 . Study guidance and sources: page 37

Use these notes during each 60-minute lesson and for revision afterward. Complete the digital homework your teacher assigns separately.

LESSON 33 | MEET THE IDEA

Quadratic zeros, factoring and equations from roots

Tuesday, February 23, 2027 . 6:00–7:00 PM Central

By the end, I can...

Solve a factorable quadratic equation, connect its roots to graph intercepts, and build a quadratic from roots and a further condition.

Last chapter, factoring changed an expression into a product. Now an **equation** asks which x -values make the two sides equal. A **root** is a solution of the equation.

The zero-product property

If $AB = 0$, then $A = 0$ or $B = 0$ (or both).

For real numbers, two nonzero factors cannot have product zero.

Use this rule only when the product equals **zero**.

Solve $x^2 - 5x + 6 = 0$

$$(x - 2)(x - 3) = 0$$

Factor: -2 and -3 have product 6 and sum -5 .

$$x - 2 = 0 \text{ or } x - 3 = 0$$

Apply the zero-product property.

$$x = 2 \text{ or } x = 3$$

Solve each linear equation.

Check in the original equation: $2^2 - 5(2) + 6 = 0$ and $3^2 - 5(3) + 6 = 0$.

For the function $f(x) = x^2 - 5x + 6$, these same numbers are its **zeros**: $f(2) = f(3) = 0$. The x -intercepts are the points $(2, 0)$ and $(3, 0)$.

LESSON 33 | KEEP EVERY POSSIBLE SOLUTION

Set the equation equal to zero first

Solve $2x^2 - 7x = 4$

$$2x^2 - 7x - 4 = 0$$

Subtract 4 from both sides before factoring.

$$(2x + 1)(x - 4) = 0$$

The middle terms are $-8x + x = -7x$.

$$x = -1/2 \text{ or } x = 4$$

Solve $2x + 1 = 0$ and $x - 4 = 0$.

Check: at $x = -1/2$, $2x^2 - 7x = 1/2 + 7/2 = 4$. At $x = 4$, it is $32 - 28 = 4$.

Solve $x^2 = 3x$ without losing a root

$$x^2 - 3x = 0$$

Subtract $3x$ from both sides.

$$x(x - 3) = 0$$

Factor out x ; do not divide by x .

$$x = 0 \text{ or } x = 3$$

Both values satisfy the original equation.

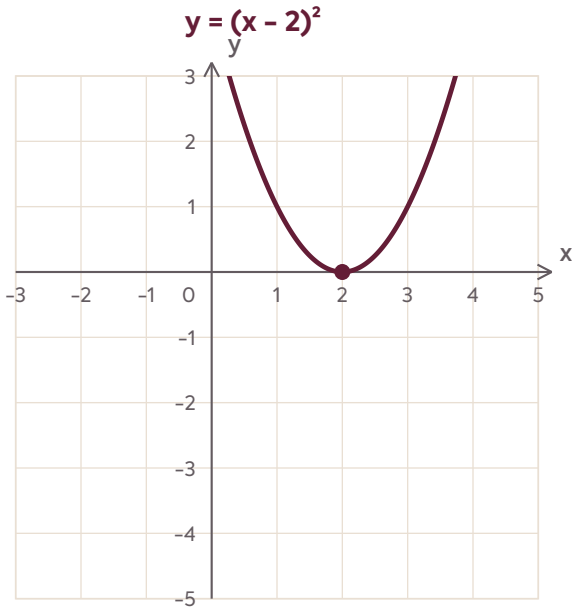
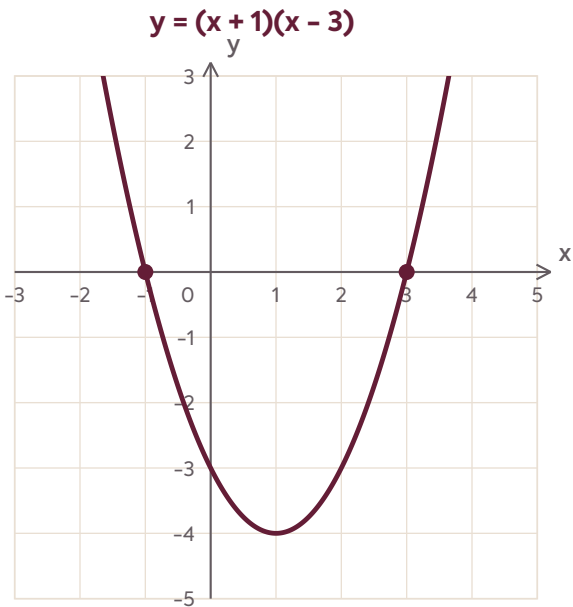
A repeated root counts as one distinct value

$$x^2 - 4x + 4 = (x - 2)^2 = 0 \text{ gives } x = 2.$$

The factor occurs twice, so 2 is a **repeated root**. There is one distinct real solution.

LESSON 33 | FACTORS, ZEROS AND INTERCEPTS

See the roots on the graph



Function	Real zeros	x-intercepts
$y = (x + 1)(x - 3)$	-1 and 3	$(-1, 0)$, $(3, 0)$
$y = (x - 2)^2$	2, a repeated zero	$(2, 0)$

The left parabola crosses the axis at each zero. The right parabola touches it at the repeated zero. The marked dots are intercepts; the curves continue beyond the graph windows.

Read the sign from the equation inside the factor

The factor $x + 1$ gives $x + 1 = 0$, so the zero is **-1**.

The factor $x - 3$ gives the zero **3**.

Do not copy the number inside the parentheses without solving.

LESSON 33 | A FURTHER POINT FIXES THE SCALE

Build a quadratic from its roots

For real roots r_1 and r_2 , write $y = a(x - r_1)(x - r_2)$, with $a \neq 0$. The roots tell you the factors. A leading coefficient or another suitable point determines a .

Roots -1 and 3; the graph passes through (0, -6)

$$y = a(x + 1)(x - 3)$$

Use one factor for each root.

$$-6 = a(0 + 1)(0 - 3)$$

Substitute the additional point.

$$-6 = -3a, \text{ so } a = 2$$

Divide by -3.

$$y = 2(x + 1)(x - 3)$$

This is the required function.

Standard form: $y = 2x^2 - 4x - 6$. At $x = 0$, $y = -6$; at $x = -1$ or 3 , $y = 0$.

Two roots alone do not choose one curve

$y = (x + 1)(x - 3)$ and $y = 2(x + 1)(x - 3)$ have the same zeros but different shapes. If a question asks for the **monic** quadratic, set $a = 1$. Otherwise retain a until another condition is given.

A repeated root r gives $y = a(x - r)^2$. To determine a from a second point, its x -coordinate must differ from the root; another intercept adds no new scale information.

LESSON 33 | SHORT CHECKS

Your turn

1. Solve $x^2 + x - 12 = 0$. Check both solutions.

2. Solve $2x^2 + 3x = 2$. Start by making one side zero.

3. For $y = (x + 2)^2$, state the zero, x-intercept and whether the root is repeated.

4. Write the monic quadratic function with roots -2 and 5 . Give factored and standard forms.

5. A graph has x-intercepts $(1, 0)$, $(4, 0)$ and passes through $(0, 8)$. Find its equation.

6. A student divides $x^2 = 5x$ by x and reports only $x = 5$. Explain the missing case.

LESSON 33 | WORKED ANSWERS

Check every root and the scale

1. $(x + 4)(x - 3) = 0$, so $x = -4$ or $x = 3$. Checks: $16 - 4 - 12 = 0$ and $9 + 3 - 12 = 0$.

2. $2x^2 + 3x - 2 = (2x - 1)(x + 2) = 0$.

$x = 1/2$ or $x = -2$. In the original equation: $1/2 + 3/2 = 2$ and $8 - 6 = 2$.

3. The zero is -2 ; the x -intercept is $(-2, 0)$. It is a **repeated root** because the factor $x + 2$ occurs twice.

4. Monic means $a = 1$: $y = (x + 2)(x - 5) = x^2 - 3x - 10$. Each given root makes a factor zero.

5. $y = a(x - 1)(x - 4)$. Using $(0, 8)$: $8 = 4a$, so $a = 2$.

$y = 2(x - 1)(x - 4) = 2x^2 - 10x + 8$.

6. Dividing by x assumes $x \neq 0$ and loses a valid solution. Instead, $x(x - 5) = 0$ gives $x = 0$ or $x = 5$. Both satisfy the original equation.

I can...

- ☐ Set an equation equal to zero before using a factored product.
- ☐ Connect a root to its x -intercept.
- ☐ Use a further condition to determine a quadratic's scale.

Next class: solve by isolating a square. Curriculum focus: A.8A, A.7B, A.6C.

LESSON 34 | MEET THE IDEA

Solving quadratics by taking square roots

Thursday, February 25, 2027 . 6:00–7:00 PM Central

By the end, I can...

Isolate a squared expression, find all its real solutions and decide which solutions make sense in a context.

Some quadratics already contain a square you can isolate. The method is: **isolate the square, take square roots, then solve the remaining linear equations.**

Solve $x^2 = 25$

$$x = \pm\sqrt{(25)}$$

Both 5 and -5 square to 25. The $\pm$ sign means plus OR minus.

$$x = 5 \text{ or } x = -5$$

List the two possible values.

A radical symbol and an equation do different jobs

$\sqrt{(25)} = 5$: the radical symbol gives the principal, nonnegative square root.

$x^2 = 25$: the equation has two solutions, ± 5 .

Do not write $\sqrt{(25)} = \pm 5$.

Solve $3x^2 = 48$

$$x^2 = 16$$

Divide both sides by 3 before taking square roots.

$$x = \pm 4$$

Check: $3(4^2) = 48$ and $3((-4)^2) = 48$.

LESSON 34 | SHIFTED SQUARES AND EXACT ANSWERS

Isolate the whole squared expression

Solve $2(x - 3)^2 = 24$

$(x - 3)^2 = 12$

$x - 3 = \pm\sqrt{12}$

$x - 3 = \pm 2\sqrt{3}$

$x = 3 \pm 2\sqrt{3}$

Exact check: for either root, $x - 3 = \pm 2\sqrt{3}$, so $2(x - 3)^2 = 2 \times 12 = 24$.

Divide both sides by 2.

Take both square roots of 12.

$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$.

Add 3 to both possibilities.

Exact solution	Approximate solution
$3 + 2\sqrt{3}$	6.46, to two decimal places
$3 - 2\sqrt{3}$	-0.46, to two decimal places

Keep exact values until the last step

$3 \pm 2\sqrt{3}$ is exact. Decimal values are rounded, so write $\approx$ when using them. Substituting a rounded value may give an answer close to 24 rather than exactly 24.

LESSON 34 | REAL SOLUTIONS AND SENSIBLE LENGTHS

Check what the square equals

Isolated square	Real solutions	Reason
$(x - h)^2 = k, k > 0$	$x = h \pm \sqrt{k}$	A positive number has two opposite real square roots.
$(x - h)^2 = 0$	$x = h$	Only zero squares to zero.
$(x - h)^2 = k, k < 0$	No real solution	A real number squared is never negative.

A square has area 75 cm^2 . Find its side length s .

$s^2 = 75$

Area of a square equals side length squared.

$s = \pm\sqrt{75} = \pm 5\sqrt{3}$

First find both algebraic possibilities.

$s = 5\sqrt{3} \text{ cm} \approx 8.66 \text{ cm}$

A side length must be positive, so use the positive value.

The equation decides the candidates; the context selects

A negative solution can be mathematically correct but unsuitable for a length. Do not reject a negative value just because it is negative: temperatures, positions and other quantities may legitimately be below zero.

LESSON 34 | SHORT CHECKS

Your turn

1. Solve $x^2 = 49$.

2. Solve $2(x + 1)^2 = 18$.

3. Solve $3(x - 2)^2 = 30$. Give exact solutions and approximations to two decimal places.

4. Solve $(x + 4)^2 = 0$.

5. A square has area 50 m^2 . Give its side length exactly and to two decimal places. Explain your root selection.

6. Does $4(x - 1)^2 + 7 = 3$ have real solutions? Explain after isolating the square.

LESSON 34 | WORKED ANSWERS

Keep both signs when they are needed

1. $x = \pm 7$. Both 7^2 and $(-7)^2$ equal 49.

2. $(x + 1)^2 = 9$, so $x + 1 = \pm 3$. Thus $x = 2$ or $x = -4$. Each makes $2(x + 1)^2$ equal 18.

3. $(x - 2)^2 = 10$, so $x = 2 \pm \sqrt{10}$.

The approximate solutions are **5.16 and -1.16**. Exact check: $3(\pm\sqrt{10})^2 = 30$.

4. $x + 4 = 0$, so $x = -4$. There is only one distinct real solution.

5. $s^2 = 50$ gives $s = \pm 5\sqrt{2}$. A side length is positive, so $s = 5\sqrt{2}$ m ≈ 7.07 m. The negative algebraic root is unsuitable for a length.

6. $4(x - 1)^2 = -4$, so $(x - 1)^2 = -1$. There is **no real solution** because a real square cannot be negative.

A reliable checking routine

Put each exact candidate back into the **original** equation. Then check the units and restrictions of any story. Do not round early or forget the second sign when the isolated square is positive.

I can...

- ☐ Isolate a square before taking square roots.
- ☐ Distinguish two, one and no real solution cases.
- ☐ Keep exact radicals and justify contextual root choices.

Next class: create a square when one is not already visible. Curriculum focus: A.8A.

LESSON 35 | MEET THE IDEA

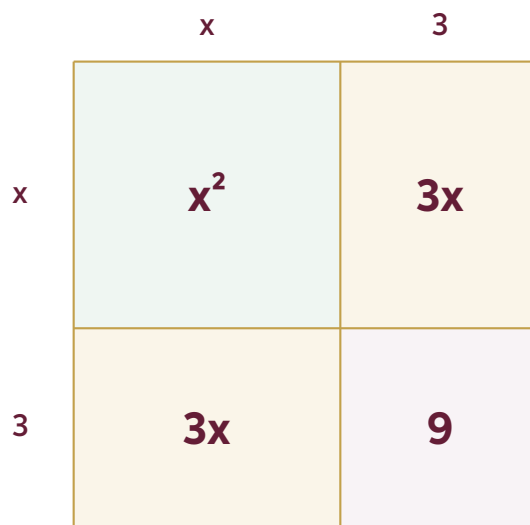
Solving quadratics by completing the square

Tuesday, March 2, 2027 . 6:00–7:00 PM Central

By the end, I can...

Create a perfect-square trinomial without changing an equation's solutions, then solve it by taking square roots.

To **complete the square**, add the missing term that makes a quadratic expression a squared binomial. The area picture explains why we halve the x-coefficient and square the result.



Two strips of area $3x$ make $6x$. The missing corner is $3 \times 3 = 9$.

$$x^2 + 6x + 9 = (x + 3)^2$$

Area illustration uses $x = 4$; the algebraic identity holds for all real x .

Half, then square

For $x^2 + bx$, add $(b/2)^2$ to make $(x + b/2)^2$.

Example: $b = 6$, half is 3, and $3^2 = 9$.

In an **equation**, add the same quantity to both sides.

LESSON 35 | A COMPLETE WORKED EXAMPLE

Keep the equation balanced

Solve $x^2 + 6x = 7$

Half of 6 is 3; $3^2 = 9$

$x^2 + 6x + 9 = 7 + 9$

$(x + 3)^2 = 16$

$x + 3 = \pm 4$

$x = 1$ or $x = -7$

Check: $1^2 + 6(1) = 7$ and $(-7)^2 + 6(-7) = 49 - 42 = 7$.

Find the term needed to complete the square.

Add 9 to both sides, preserving equality.

Rewrite the left as a perfect square.

Use both square roots of 16.

Subtract 3 from each possibility.

Expression	Half the x-coefficient	Term to add	Squared form
$x^2 + 8x$	4	16	$(x + 4)^2$
$x^2 - 4x$	-2	4	$(x - 2)^2$
$x^2 + 5x$	5/2	25/4	$(x + 5/2)^2$

A negative half-coefficient still gives a positive added square. A fractional half-coefficient is allowed: $(5/2)^2 = 25/4$.

LESSON 35 | A SIMPLE NONMONIC EQUATION

Make the leading coefficient 1 first

Solve $2x^2 + 8x - 6 = 0$

$$x^2 + 4x - 3 = 0$$

$$x^2 + 4x = 3$$

$$x^2 + 4x + 4 = 3 + 4$$

$$(x + 2)^2 = 7$$

$$x = -2 \pm \sqrt{7}$$

Divide EVERY term on both sides by 2.

Move the constant to the other side.

Half of 4 is 2; add $2^2 = 4$ to both sides.

The left side is now a square.

Take square roots, then subtract 2.

Check using an equivalent form

$$2x^2 + 8x - 6 = 2[(x + 2)^2 - 7].$$

For either root, $(x + 2)^2 = 7$, so the original expression becomes $2(7 - 7) = 0$.

Do not halve the original coefficient 8 while leaving the leading coefficient 2 in place. First divide the whole equation by 2, then use the new coefficient 4.

LESSON 35 | REWRITE A FUNCTION WITHOUT CHANGING IT

The same idea reveals the vertex

For a function rule, keep the expression equivalent by **adding and subtracting** the same number inside it. This adds zero overall.

Rewrite $y = x^2 + 6x + 2$ in vertex form

$y = x^2 + 6x + 9 - 9 + 2$

$y = (x + 3)^2 - 7$

Add and subtract 9, the square of half of 6.

The square reveals the vertex $(-3, -7)$.

Your task	What preserves the value or equality?
Solve $x^2 + 6x = 7$	Add 9 to BOTH sides: $x^2 + 6x + 9 = 16$.
Rewrite $x^2 + 6x + 2$	Add 9 and subtract 9 within the same expression.

A common error to catch

$x^2 + 6x = 7$ does **not** become $(x + 3)^2 = 7$.

Expanding that square adds 9 to the left side. The right side must also gain 9, giving 16.

Completing the square always creates an equivalent equation when you use balanced operations. The square-root step then tells you whether there are two, one or no real solutions.

LESSON 35 | SHORT CHECKS

Your turn

1. Fill the blank and factor: $x^2 + 10x + \underline{\hspace{2cm}}$.

2. Solve $x^2 + 4x = 12$ by completing the square.

3. Solve $x^2 - 6x = 7$ by completing the square.

4. Solve $2x^2 + 4x - 4 = 0$. Divide first, then complete the square.

5. A student changes $x^2 + 8x = 5$ into $(x + 4)^2 = 5$. Correct the equation and finish solving.

LESSON 35 | WORKED ANSWERS

Check the added term on both sides

1. Half of 10 is 5, and $5^2 = 25$. Thus $x^2 + 10x + 25 = (x + 5)^2$.

2. Add 4 to both sides: $x^2 + 4x + 4 = 16$.

$(x + 2)^2 = 16$, so $x + 2 = \pm 4$ and $x = 2$ or -6 .

Checks: $4 + 8 = 12$ and $36 - 24 = 12$.

3. Add 9: $x^2 - 6x + 9 = 16$.

$(x - 3)^2 = 16$, so $x = 7$ or -1 .

Checks: $49 - 42 = 7$ and $1 + 6 = 7$.

4. Divide by 2: $x^2 + 2x = 2$. Add 1: $(x + 1)^2 = 3$.

$x = -1 \pm \sqrt{3}$. Check the original as $2[(x + 1)^2 - 3] = 2(3 - 3) = 0$.

5. Add 16 to **both** sides: $(x + 4)^2 = 21$.

Then $x = -4 \pm \sqrt{21}$. Check: $x^2 + 8x = (x + 4)^2 - 16 = 21 - 16 = 5$.

I can...

- ☐ Make the leading coefficient 1 before using the half-coefficient rule.
- ☐ Add the same square to both sides of an equation.
- ☐ Connect a completed square to vertex form.

Next class: the quadratic formula and real-root decisions. Curriculum focus: A.8A.

LESSON 36 | MEET THE IDEA

The quadratic formula and real-root decisions

Thursday, March 4, 2027 . 6:00–7:00 PM Central

By the end, I can...

Use the quadratic formula with correct signs and parentheses, identify the number of real roots and write real linear factors when possible.

First write $ax^2 + bx + c = 0$, with $a \neq 0$. Completing the square on this general equation produces the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The **whole numerator** is divided by $2a$. The $\pm$ gives a plus case and a minus case. A missing x -term means $b = 0$; a missing constant means $c = 0$.

Solve $2x^2 - 3x - 2 = 0$

$$a = 2, b = -3, c = -2$$

$$b^2 - 4ac = 9 + 16 = 25$$

$$x = (3 \pm 5)/4$$

$$x = 2 \text{ or } x = -1/2$$

Include the signs of the coefficients.

$$(-3)^2 - 4(2)(-2) = 25.$$

$$-b = 3 \text{ and } 2a = 4.$$

Compute $(3 + 5)/4$ and $(3 - 5)/4$ separately.

LESSON 36 | AN IRRATIONAL-ROOT EXAMPLE

Keep radicals exact until the end

Solve $2x^2 - 4x - 1 = 0$

$$a = 2, b = -4, c = -1$$

$$D = (-4)^2 - 4(2)(-1) \\ = 24$$

$$x = (4 \pm \sqrt{(24)})/4$$

$$x = (4 \pm 2\sqrt{(6)})/4$$

$$x = (2 \pm \sqrt{(6)})/2$$

The equation is already in standard form.

The expression under the radical is called the discriminant, D .

Substitute $-b = 4$ and $2a = 4$.

Simplify $\sqrt{(24)}$ as $2\sqrt{(6)}$.

Divide EVERY term of the numerator and denominator by 2.

These are approximately **2.22** and **-0.22**. Keep the radical forms for exact work.

Check without messy decimal arithmetic

The roots can also be written $x = 1 \pm \sqrt{(6)}/2$.

Then $(x - 1)^2 = 6/4 = 3/2$.

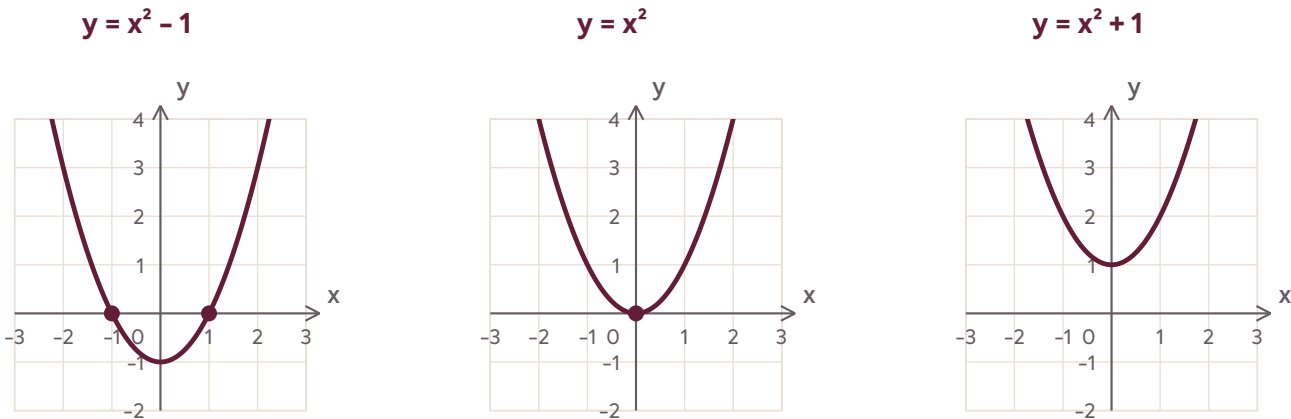
The original expression is $2(x - 1)^2 - 3$, which becomes $2(3/2) - 3 = 0$.

An equation that does not factor neatly over the integers may still have two real solutions. The formula finds them without guessing an integer factor pair.

LESSON 36 | TWO, ONE OR NO REAL ROOTS

The discriminant tells you what to expect

For $ax^2 + bx + c = 0$, calculate $D = b^2 - 4ac$. Its sign tells whether the square-root part of the formula is positive, zero or not real.



Discriminant	Distinct real roots	Graph of $y = ax^2 + bx + c$
$D > 0$	Two	Crosses the x-axis twice.
$D = 0$	One, repeated	Touches the x-axis at the vertex.
$D < 0$	None	Does not meet the x-axis.

In the graphs above, the discriminants are 4, 0 and -4, respectively. The curves continue outside the windows.

Stay in the real-number scope

A negative discriminant means **no real solutions**. Do not write $\sqrt{(-4)} = -2$. A real square cannot be negative. Complex-number calculations are outside this lesson.

LESSON 36 | FINISH THE LINK FROM CHAPTER 5

Real roots give real linear factors

If a quadratic has real roots r_1 and r_2 , its expression is $a(x - r_1)(x - r_2)$. Keep the original leading coefficient a .

Factor $x^2 - 2x - 1$ over the real numbers

$$D = (-2)^2 - 4(1)(-1) \\ = 8$$

No integer factor pair is needed. Use the formula.

$$x = (2 \pm \sqrt{8})/2 \\ = 1 \pm \sqrt{2}$$

These are the two roots.

$$[x - (1 + \sqrt{2})] \\ \times [x - (1 - \sqrt{2})]$$

Use one linear factor for each root. Here $a = 1$.

Verify with a difference of squares

$$[(x - 1) - \sqrt{2}][(x - 1) + \sqrt{2}] \\ = (x - 1)^2 - 2 \\ = x^2 - 2x - 1.$$

If the original expression were $3x^2 - 6x - 3$, the same roots would apply, but the factorization would need **3 times** the product above. Roots alone do not supply the leading coefficient.

When $D = 0$, the form is $a(x - r)^2$. When $D < 0$, there are no real linear factors; this does not prevent taking out a common constant factor.

LESSON 36 | SHORT CHECKS

Your turn

Use the quadratic formula for Questions 1–3, even if another method also works. Give exact answers.

1. $x^2 - 3x - 4 = 0$

2. $2x^2 + 4x - 1 = 0$

3. $x^2 - 6x + 9 = 0$

4. Use the discriminant to classify the real roots of $x^2 + 2x + 5 = 0$.

5. Find the roots of $x^2 - 4x + 1 = 0$, then factor $x^2 - 4x + 1$ over the real numbers.

6. For $2x^2 - 3x - 2 = 0$, a student gets $x = (3 \pm 5)/2$. Identify the error and give the corrected roots.

LESSON 36 | WORKED ANSWERS

Check the signs and the denominator

1. $a = 1$, $b = -3$, $c = -4$; $D = 9 + 16 = 25$.

$x = (3 \pm 5)/2$, so $x = 4$ or -1 . Checks: $16 - 12 - 4 = 0$ and $1 + 3 - 4 = 0$.

2. $a = 2$, $b = 4$, $c = -1$; $D = 16 + 8 = 24$.

$x = (-4 \pm 2\sqrt{(6)})/4 = (-2 \pm \sqrt{(6)})/2$.

Check: $2x^2 + 4x - 1 = 2(x + 1)^2 - 3$; each root makes $(x + 1)^2 = 3/2$.

3. $D = 36 - 36 = 0$. $x = (6 \pm 0)/2 = 3$, one repeated real root. Check: $9 - 18 + 9 = 0$.

4. $D = 4 - 20 = -16$, so there are **no real roots**.

5. $D = 16 - 4 = 12$; $x = (4 \pm 2\sqrt{(3)})/2 = 2 \pm \sqrt{(3)}$.

Factors: $[x - (2 + \sqrt{(3)})][x - (2 - \sqrt{(3)})]$.

Check: $(x - 2)^2 - 3 = x^2 - 4x + 1$.

6. The denominator must be $2a = 4$, not 2.

$x = (3 \pm 5)/4$ gives $x = 2$ or $-1/2$.

I can...

- ☐ Identify a , b and c with their signs.
- ☐ Use the discriminant to classify real-root cases.
- ☐ Keep the scale factor when constructing real linear factors.

Next class: fit and interpret a quadratic model from data. Curriculum focus: A.8A, A.10E.

LESSON 37 | MEET THE IDEA

Quadratic regression and real-world models

Tuesday, March 9, 2027 . 6:00–7:00 PM Central

By the end, I can...

Enter paired data into a graphing tool, fit a quadratic, interpret its vertex and predictions, and explain the limits of the model.

These **fictional data** describe height after an object is released. Time t is measured in seconds from release; height is measured in meters above the ground. The slight variation gives us practice fitting a curve.

Time t (seconds)	Height (meters)
0	1.05
0.5	2.15
1	2.95
1.5	3.45
2	2.95
2.5	2.15
3	1.05

Your task includes using technology

Enter the data and run a quadratic regression yourself. Record the fitted coefficients, display the curve with the points, and use your result to answer questions. The following pages guide the process.

A scatterplot helps you judge whether a quadratic is plausible. A curved pattern alone does not prove that a quadratic is the right model for every situation.

LESSON 37 | A DESMOS REGRESSION WORKFLOW

Fit the curve yourself

Open a fresh [Desmos Graphing Calculator](#). Use the teacher’s approved alternative if needed; choose a quadratic model $ax^2 + bx + c$.

Step	What to do
1. Enter	Create a table with columns x_1 and y_1 . Enter time in x_1 and height in y_1 , one pair per row.
2. Fit	On a new expression line, type the regression below. The tilde $\sim$ asks for a fit instead of an exact equation.
3. Record	Read and record the fitted values of a , b and c . Avoid assigning these letters values before fitting.
4. Inspect	Use Zoom Fit to see the data and curve. Check that all seven data pairs were entered correctly.
5. Predict	Define $h(x) = ax^2 + bx + c$ using the fitted parameters. Evaluate $h(1.25)$ and find the vertex.

Regression to enter

$y_1 \sim ax_1^2 + bx_1 + c$

Keyboard form: $y_1 \sim a x_1^2 + b x_1 + c$

The subscript 1 refers to the table columns. The x in $h(x)$ represents time.

My fitted values: $a =$ _____ $b =$ _____ $c =$ _____

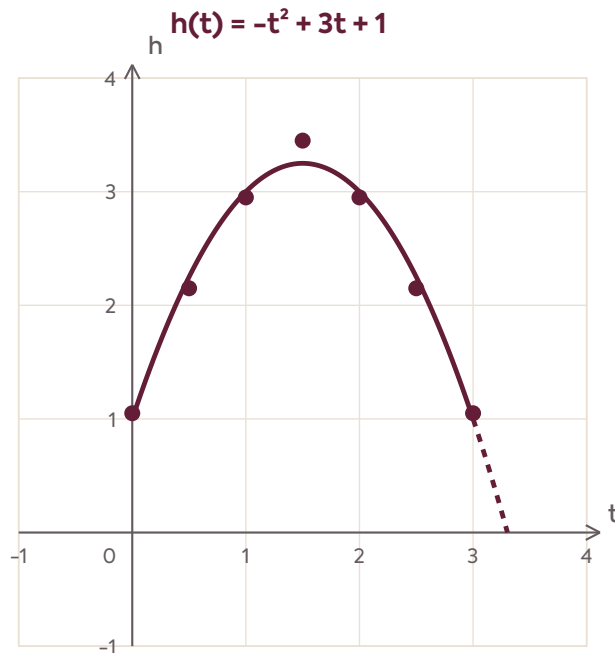
My predicted $h(1.25)$: _____ meters. Compare with the worked model next.

Workflow checked September 9, 2026. Official help: [Desmos regressions guide](#).

LESSON 37 | A MODEL IS AN ESTIMATE

Read what the fitted curve says

The fitted coefficients are approximately $a = -1$, $b = 3$, $c = 1$. Use the displayed model $h(t) = -t^2 + 3t + 1$ for the calculations in these notes.



Dots: the supplied data.

Solid curve: the fitted model over $0 \leq t \leq 3$.

Short dashed part: a prediction beyond the data.

t is measured in seconds.

h is measured in meters.

Find the model's highest point

$$h(t) = -(t - 1.5)^2 + 3.25$$

Vertex: (1.5, 3.25)

Rewrite in vertex form by completing the square.

The fitted maximum is 3.25 meters at 1.5 seconds.

The data value at $t = 1.5$ is 3.45 meters, slightly above the fitted curve. **Residual = observed value - predicted value:** $3.45 - 3.25 = 0.20$ meters. A best-fit curve need not pass through every point.

Over the observed time interval $0 \leq t \leq 3$, the displayed model has range $1 \leq h \leq 3.25$. That describes the model's outputs, not the minimum and maximum of the noisy data.

LESSON 37 | WITHIN AND BEYOND THE DATA

Predict with a sensible boundary

Estimate the height at 1.25 seconds

$$h(1.25) = -(1.25)^2 + 3(1.25) + 1$$

Substitute the time into the fitted model.

$$h(1.25) = 3.1875$$

The estimated height is about 3.19 meters.

This is interpolation: 1.25 is within the observed interval from 0 to 3 seconds.

Predict when the height reaches ground level

$$-t^2 + 3t + 1 = 0$$

Ground level means $h = 0$.

$$t^2 - 3t - 1 = 0$$

Multiply both sides by -1 .

$$t = (3 \pm \sqrt{(13)})/2$$

Use the quadratic formula.

$$t \approx 3.30 \text{ or } t \approx -0.30$$

Time starts at release, so reject the negative root.

Label this landing time as an extrapolation

The latest data point is at 3 seconds. A landing time of about 3.30 seconds goes beyond the data. It is a tentative prediction, assuming the model remains suitable until landing. Do not treat it as an observed fact.

If that assumption is made, the flight interval is $0 \leq t \leq (3 + \sqrt{(13)})/2$ and the modeled height range is $0 \leq h \leq 3.25$. Do not extend this flight model to negative heights after landing.

LESSON 37 | USE YOUR FITTED MODEL

Your turn

Use the supplied dataset and the graphing tool. Keep unrounded coefficients in the tool; round reported heights and times to two decimal places where appropriate.

1. Run a quadratic regression and record a , b and c . Show the data points and fitted curve on the same graph.

2. Give the model's maximum height and the time it occurs. How does this differ from the largest supplied height value?

3. Predict $h(1.25)$. Is this interpolation or extrapolation? Explain.

4. Use the displayed model to find when the height is 3 meters. Explain why there are two times.

5. Find the predicted landing time. Explain which root you reject, why the prediction extends beyond the data, and why $h(10)$ is not a sensible flight prediction.

LESSON 37 | WORKED ANSWERS

Check the calculation and its meaning

1. $a \approx -1$, $b \approx 3$, $c \approx 1$, giving $h(t) = -t^2 + 3t + 1$. Your plot should show seven points close to a downward-opening parabola. A different result is a reason to check data entry and the regression expression.
2. Vertex form is $h(t) = -(t - 1.5)^2 + 3.25$. The **fitted maximum is 3.25 meters at 1.50 seconds**. The largest supplied value is **3.45 meters**, also at 1.50 seconds; the model does not match every observation exactly.
3. $h(1.25) = -1.5625 + 3.75 + 1 = 3.1875$, or **about 3.19 meters**. It is **interpolation** because 1.25 lies between 0 and 3 seconds.
4. $-t^2 + 3t + 1 = 3$ gives $t^2 - 3t + 2 = 0$.
 $(t - 1)(t - 2) = 0$, so **$t = 1$ or 2 seconds**. The model reaches 3 meters once while rising and once while falling.
5. The roots are $(3 \pm \sqrt{13})/2$. Reject about -0.30 seconds because time begins at release. The possible landing time is **about 3.30 seconds**, beyond the observed 0–3 second interval. Also, $h(10) = -100 + 30 + 1 = -69$ meters; extending this flight model far past landing is inappropriate.

A close fit is not a guarantee

For these data, R^2 is about 0.987. It summarizes how closely this fitted model matches these data; it does not establish cause and effect or guarantee accurate extrapolation. Always inspect the graph, units and context.

Curriculum focus: A.8B. Next class: compare the four solving methods and review your evidence before the Unit 6 checkpoint.

LESSON 38 | MEET THE IDEA

Unit 6 checkpoint and choosing a quadratic method

Thursday, March 11, 2027 . 6:00–7:00 PM Central

By the end, I can...

Choose and justify a quadratic-solving method, check the resulting solutions and explain what a model can reasonably tell me.

This is a **revision and correction guide**. The graded Unit 6 checkpoint is supplied separately by your teacher. No new topic is introduced today.

Method	A useful starting form	Why choose it?
Factoring	$x^2 - 5x = 0$	A common factor or familiar trinomial gives a quick zero product.
Square roots	$(x - 2)^2 = 7$	A squared expression is already isolated, or easy to isolate.
Complete the square	$x^2 - 4x = 1$	Create a squared binomial; it also reveals vertex form.
Quadratic formula	$3x^2 + 2x - 2 = 0$	Use a general method when factoring is not convenient.

Our one-hour checkpoint lesson

- 5 minutes: retrieve key ideas.
- 20 minutes: the Unit 6 checkpoint.
- 25 minutes: corrections and focused reteaching.
- 10 minutes: exit check and next steps.

Your teacher uses the checkpoint together with your work from Classes 33–37. One short assessment cannot demonstrate every skill by itself.

LESSON 38 | A SHORT CHECKING ROUTINE

Use the error to choose a repair

Error to notice	Repair
Dividing $x^2 = 5x$ by x	Factor $x(x - 5) = 0$ to retain the possible root $x = 0$.
Reporting only the positive square root	If the isolated square equals a positive number, use both $\pm$ cases. Then apply any context restrictions.
Adding only to one side	Complete the square with balanced operations on both sides.
Using 2 instead of $2a$	Keep the whole numerator over the denominator $2a$.
Giving a negative flight time	Check where the story starts and which root lies in its domain.
Treating a fitted curve as an exact law	Inspect the data, residuals and interval; label extrapolations as predictions.

Check in three steps

- 1. **Algebra:** substitute each candidate into the original equation.
- 2. **Completeness:** did you lose a zero root or forget a second sign?
- 3. **Meaning:** are the units, time interval and selected roots sensible?

My correction log

The error I made: _____
The rule I need: _____
A lesson and new question to retry: _____

LESSON 38 | PRACTICE, NOT THE GRADED CHECKPOINT

A short revision check

Use the named method so you revisit all four. Check solutions and explain your choice when there is a context.

1. Factoring: solve $x^2 - 5x = 0$.

2. Square roots: solve $(x - 2)^2 = 7$.

3. Completing the square: solve $x^2 - 4x - 1 = 0$.

4. Quadratic formula: solve $3x^2 + 2x - 2 = 0$.

5. A fictional height model is $h(t) = -t^2 + 2t + 3$, fitted to data from 0 to 2 seconds after release. Predict when $h = 0$. Which root is relevant, and is this interpolation or extrapolation?

LESSON 38 | WORKED ANSWERS

Check, correct and take the next step

1. $x(x - 5) = 0$, so $x = 0$ or 5 . Checks: $0 - 0 = 0$ and $25 - 25 = 0$.

2. $x - 2 = \pm\sqrt{(7)}$, so $x = 2 \pm \sqrt{(7)}$. Either value gives $(x - 2)^2 = 7$.

3. $x^2 - 4x = 1$. Add 4: $(x - 2)^2 = 5$.

$x = 2 \pm \sqrt{(5)}$. Check the original as $(x - 2)^2 - 5 = 0$.

4. $a = 3$, $b = 2$, $c = -2$; $D = 4 + 24 = 28$.

$x = (-2 \pm 2\sqrt{(7)})/6 = (-1 \pm \sqrt{(7)})/3$.

To check, use $3x^2 + 2x - 2 = [(3x + 1)^2 - 7]/3$. Each root makes the numerator 0.

5. $-t^2 + 2t + 3 = 0$ gives $(t - 3)(t + 1) = 0$ after multiplying by -1 . The roots are 3 and -1 . Use **3 seconds**; -1 is before release. It is **extrapolation** beyond the observed 0–2 second interval, so the prediction needs caution.

Choose your next practice using evidence

Class 33: roots and factors. Class 34: square-root signs and context.

Class 35: balanced completion of the square. Class 36: formula signs and real-root cases. Class 37: fitting and interpreting a model.

Spring break follows. Chapter 7 begins Tuesday, March 23, 2027: arithmetic sequences and their rules. Curriculum focus: A.8A–B, A.7B and A.6C.

KEEP THIS PAGE FOR REVISION

Chapter 6 quick reference

Idea	Remember
Zero product	$AB = 0$ means $A = 0$ or $B = 0$. Make one side zero before applying the rule.
Roots and factors	Real roots r_1, r_2 give $a(x - r_1)(x - r_2)$, with $a \neq 0$. Keep the leading coefficient.
Square-root method	After isolating $(x - h)^2 = k$: if $k > 0$, $x = h \pm \sqrt{k}$; if $k = 0$, $x = h$; if $k < 0$, no real solutions.
Complete the square	Make the x^2 -coefficient 1. For $x^2 + bx$, add $(b/2)^2$ to both sides of the equation.
Quadratic formula	For $ax^2 + bx + c = 0$: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ The denominator divides the whole numerator.
Discriminant	$D = b^2 - 4ac$. Positive: two distinct real roots; zero: one repeated real root; negative: no real roots.
Quadratic regression	Fit a model to paired data with technology. Check the curve, units, data entry and contextual restrictions.
Prediction limits	Interpolation is within the observed input interval. Extrapolation goes beyond it and needs stronger justification.
Final check	Substitute each candidate into the original equation. Then apply the context. Keep exact radicals until rounding is needed.

These methods work together. Factoring can be quick; square roots exploit a visible square; completing the square reveals structure; the formula gives a general route and identifies real-root cases.

STUDY GUIDANCE AND CURRICULUM REFERENCES

Make these notes work for you

A small routine for each lesson

Before: recall the factor or square pattern you will need.

During: predict the next worked step and explain why it is valid.

After: attempt the study checks with answers covered, then do the assigned digital homework.

Study checks and graded assessments

The PDFs include answers to their own revision questions. The Class 36 quiz and Class 38 graded checkpoint are separate teacher-provided materials. Their unpublished questions and answer keys are not included.

Graphs, diagrams and data

Graphs are drawn from the stated equations and use labeled axes. Unrestricted curves continue beyond the windows. The completing-square area picture uses positive lengths; its algebraic identity also holds for other real x . All data and numerical stories are fictional teaching examples.

In Class 37, students must perform the regression step themselves. The fitted curve is solid over the supplied time interval and dashed beyond it. Predictions from the displayed equation are model estimates, not observed outcomes.

Curriculum alignment

Standard Algebra I, §111.39(c): A.6C, A.7B, A.8A–B and A.10E, with earlier algebra and graph skills revisited. Advanced Grade 8 Algebra I, §111.31(d): corresponding (6)(C), (7)(B), (8)(A)–(B) and (10)(E).

These are original Cambridge Veritas supplementary notes, not an official TEA publication. The dates follow our agreed course calendar, not state-required pacing. All solving stays within the real-number scope.

Official references

Texas Education Agency: [Standard Algebra I, §111.39, mathematics TEKS](#).

Texas Education Agency: [Advanced Grade 8 Algebra I, §111.31, adopted 2025](#).

Desmos: [Official regression instructions](#).

Standards and software guidance checked September 9, 2026. Chapter 6 edition: 1.0.