

LESSON 25 | MEET THE IDEA

Dividing linear and quadratic polynomials

Tuesday, January 26, 2027 . 6:00–7:00 PM Central

By the end, I can...
Divide linear and quadratic polynomials, express any remainder and keep the original divisor nonzero.

Division asks: **what multiplies the divisor to make the dividend?** If it does not divide exactly, keep the leftover part as a remainder.

Word	Meaning	In $17 \div 5$
Dividend	What you divide	17
Divisor	What you divide by	5
Quotient	The whole-number part here	3
Remainder	What is left after multiplying back	2

$17 = 5 \times 3 + 2$, so $17/5 = 3 + 2/5$.
For polynomials, the same structure is **dividend = divisor $\times$ quotient + remainder**.

Divide by a monomial | $(6x^2 + 9x) / (3x)$, $x \neq 0$

$6x^2/(3x) + 9x/(3x)$
 $2x + 3$
Keep $x \neq 0$, because the original divisor is $3x$.

Divide each numerator term by the same divisor.
Check: $3x(2x + 3) = 6x^2 + 9x$.

Long division, one step at a time

Divide $x^2 + 5x + 6$ by $x + 2$. The original divisor requires $x \neq -2$. Line up matching powers before you subtract.

	x	$+ 3$	Quotient: $x + 3$
$x + 2$	$\begin{array}{r} \overline{) x^2 + 5x + 6} \\ - x^2 \\ \hline 3x + 6 \\ - 3x + 6 \\ \hline 0 \end{array}$		Dividend: $x^2 + 5x + 6$
	$\begin{array}{r} \\ - x^2 \\ \hline 3x + 6 \end{array}$		Subtract $x(x + 2)$.
	$\begin{array}{r} \\ \\ 3x + 6 \end{array}$		Bring down $+6$.
	$\begin{array}{r} \\ \\ 3x + 6 \\ - 3x + 6 \\ \hline 0 \end{array}$		Subtract $3(x + 2)$.
	$\begin{array}{r} \\ \\ + 6 \\ - + 6 \\ \hline 0 \end{array}$		Remainder: 0

Step	Reason
$x^2 \div x = x$	Choose x as the first term of the quotient.
Subtract $x(x + 2) = x^2 + 2x$	This removes the leading x^2 term and leaves $3x + 6$.
$3x \div x = 3$	Choose $+3$ as the next quotient term.
Subtract $3(x + 2) = 3x + 6$	The remainder is 0, so the division is exact.

Result: $x + 3$, with $x \neq -2$. Multiplying $(x + 2)(x + 3)$ gives the original $x^2 + 5x + 6$. The restriction remains even when the quotient has no denominator.

LESSON 25 | WHEN DIVISION IS NOT EXACT

A remainder still belongs in the answer

Divide $x^2 + 3x + 5$ by $x + 1$, with $x \neq -1$

$$x^2 \div x = x$$

The first quotient term is x .

$$\begin{array}{r} (x^2 + 3x + 5) \\ - (x^2 + x) \\ \hline 2x + 5 \end{array}$$

Multiply $x(x + 1)$, then subtract the whole product.

$$2x \div x = 2$$

The next quotient term is $+2$.

$$(2x + 5) - (2x + 2) = 3$$

Subtract $2(x + 1)$. The remainder is 3 .

Write quotient plus remainder divided by divisor

$$(x^2 + 3x + 5)/(x + 1) = x + 2 + 3/(x + 1), x \neq -1.$$

The remainder is not just an extra $+3$; it is divided by the original divisor.

Check by rebuilding the dividend

$$\begin{aligned} &(x + 1)(x + 2) + 3 \\ &= x^2 + 3x + 2 + 3 \\ &= x^2 + 3x + 5. \text{ The original dividend is recovered.} \end{aligned}$$

Stop when the remainder is zero or its degree is less than the divisor's degree. Here the remainder has degree 0 and the divisor has degree 1.

LESSON 25 | CONSTANT QUOTIENTS AND REMINDERS

When the degrees are equal

Linear ÷ linear | $(5x + 7)/(x + 1)$, $x \neq -1$

$$5x \div x = 5$$

The quotient is the constant 5.

$$(5x + 7) - 5(x + 1) = 2$$

The remainder is 2.

$$5 + 2/(x + 1)$$

Write the remainder over the original divisor.

Quadratic ÷ quadratic | $(3x^2 + 2x + 5)/(x^2 + 1)$

$$3x^2 \div x^2 = 3$$

The quotient is the constant 3.

$$\begin{array}{r} (3x^2 + 2x + 5) \\ - 3(x^2 + 1) \\ \hline \end{array} = 2x + 2$$

Subtract the entire product. The remainder has degree 1.

$$3 + (2x + 2)/(x^2 + 1)$$

The remainder degree is lower than the divisor degree, so stop.

For real x , $x^2 + 1$ is always positive. There are no excluded real x -values for this divisor.

Three reminders

Missing term: write $x^2 + 0x - 3$ to keep columns aligned.

Remainder: a polynomial remainder may be negative.

Cancellation: cancel common factors, not separate terms in sums. For example, $(x + 2)/x = 1 + 2/x$, not 2, with $x \neq 0$.

LESSON 25 | SHORT CHECKS

Your turn

Give a quotient plus remainder/divisor when needed. State which real x -values, if any, the original divisor excludes.

1. Divide $(4x + 9)$ by $(x + 2)$.

2. Divide $(x^2 + 7x + 12)$ by $(x + 3)$.

3. Divide $(x^2 - 3)$ by $(x + 1)$. Write the missing $0x$ term first.

4. Divide $(2x^2 + x + 4)$ by $(x^2 + 2)$.

5. Simplify $(6x^2 - 3x)/(3x)$. Verify your answer by multiplication and retain the restriction.

LESSON 25 | WORKED ANSWERS

Multiply back to check

1. Quotient 4, remainder 1: $4 + 1/(x + 2)$, $x \neq -2$. Check: $4(x + 2) + 1 = 4x + 9$.
2. Subtract $x(x + 3)$ to leave $4x + 12$; subtract $4(x + 3)$ to leave 0. Result: $x + 4$, $x \neq -3$. Check: $(x + 3)(x + 4) = x^2 + 7x + 12$.
3. Start with $x^2 + 0x - 3$. Subtract $x(x + 1)$ to leave $-x - 3$. The next quotient term is -1 ; subtract $-1(x + 1)$ to leave -2 . Result: $x - 1 - 2/(x + 1)$, $x \neq -1$. Check: $(x + 1)(x - 1) - 2 = x^2 - 3$.
4. The quotient is 2; subtract $2(x^2 + 2)$ to leave x . Result: $2 + x/(x^2 + 2)$. There are **no excluded real values**, because $x^2 + 2 \geq 2$. Check: $2(x^2 + 2) + x = 2x^2 + x + 4$.
5. $2x - 1$, with $x \neq 0$. Check: $3x(2x - 1) = 6x^2 - 3x$. The original fraction is undefined at $x = 0$, even though $2x - 1$ is defined there.

The final check has two parts

Identity: divisor $\times$ quotient + remainder = dividend.

Domain: the original divisor must not be zero. A correct expansion does not remove an excluded value.

I can...

- ☐ Divide, multiply, subtract and repeat in aligned columns.
- ☐ Interpret a remainder and know when to stop.
- ☐ Retain the original divisor restrictions.

Next class: review and the Chapter 4 checkpoint. Curriculum focus: A.10C.