



CHAPTER 07

Algebra I

Sequences and exponential functions

Classes 39–43 . March 23 – April 6, 2027
Tuesdays and Thursdays . 6:00–7:00 PM Central

NOTICE THE CHANGE. EXPLAIN THE PATTERN.

$$a_n = a_1 + (n - 1)d$$

Add the same amount at each step.

$$a_n = a_1 r^{n-1}$$

Multiply by the same factor at each step.

$$f(t) = ab^t$$

Model a repeated percentage change.

Clear worked steps . Discrete and continuous graphs
Technology practice . Worked answers . Chapter revision

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Your route through Chapter 7

Start with a sequence, build a rule, then turn repeated multiplication into a model. Each lesson includes short study checks and worked answers.

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Chapter quick reference: page 33 . Study guidance and sources: page 34

Use these notes during each 60-minute class and for revision afterward. Complete digital homework separately in the dashboard. Your teacher supplies graded quizzes and checkpoint materials separately.

LESSON 39 | MEET THE IDEA

Arithmetic sequences: recursive and explicit rules

Tuesday, March 23, 2027 . 6:00–7:00 PM Central

By the end, I can...

Recognize an arithmetic sequence, write both kinds of rule and connect its discrete graph to a linear relationship.

A **sequence** is an ordered list. Each number is a **term**. The term number tells you a position, not a value. In this lesson we number terms **$n = 1, 2, 3, \dots$**

Term number n	1	2	3	4
Term value a_n	8	13	18	23

Same amount added each time

$13 - 8 = 5$, $18 - 13 = 5$ and $23 - 18 = 5$.

An **arithmetic sequence** has the same difference between consecutive terms. Here the **common difference** is $d = 5$.

Read a_n as "a sub n": the value of term n . Here $a_1 = 8$, $a_2 = 13$ and $a_4 = 23$. The little number is a position label, not an exponent.

Check more than one jump

A short list suggests a pattern; it does not prove how every later term must behave. Use the stated rule or the assumption that the sequence is arithmetic before extending it.

LESSON 39 | START WITH THE INDEX

Two rules, one sequence

Recursive: use the previous term

For 8, 13, 18, 23, ...:
 $a_1 = 8; a_n = a_{n-1} + 5$, for $n = 2, 3, \dots$
The starting value is essential. "Add 5" alone does not tell you where to begin.

Explicit: go straight to any term

$a_n = a_1 + (n - 1)d$
 $a_n = 8 + 5(n - 1)$
 $a_{10} = 8 + 5(9) = 53$

Starting at term 1, reaching term n takes $n - 1$ jumps.
Use $a_1 = 8$ and $d = 5$; $n = 1, 2, 3, \dots$.
Term 10 is nine jumps after the first term.

Check the first term: $8 + 5(1 - 1) = 8$. An explicit rule uses the index directly; it does not require calculating earlier terms.

Rule type	What must be supplied?	Best use
Recursive	First term and a rule using the previous term	Generate terms in order.
Explicit	Formula in n and the allowed indices	Find a distant term directly.

Avoid the extra jump: $8 + 5n$ gives 13 when $n = 1$, so it is not the rule for this sequence with first index 1.

LESSON 39 | DECREASE AND CONTEXT

A difference can be negative

50, 43, 36, 29, ... is arithmetic

$$d = 43 - 50 = -7$$

Subtract 7 each time; adding -7 means the same thing.

$$a_n = 50 - 7(n - 1)$$

The first term is at $n = 1$.

$$a_6 = 50 - 35 = 15$$

There are five steps from term 1 to term 6.

A theater has 12 seats in row 1 and 4 more in each next row.

$$s_n = 12 + 4(n - 1)$$

n is a whole-number row label: 1, 2, 3,

$$s_8 = 12 + 4(7) = 40$$

Row 8 contains 40 seats.

$$s_1 = 12; s_n = s_{n-1} + 4$$

This recursive version applies for $n = 2, 3, \dots$

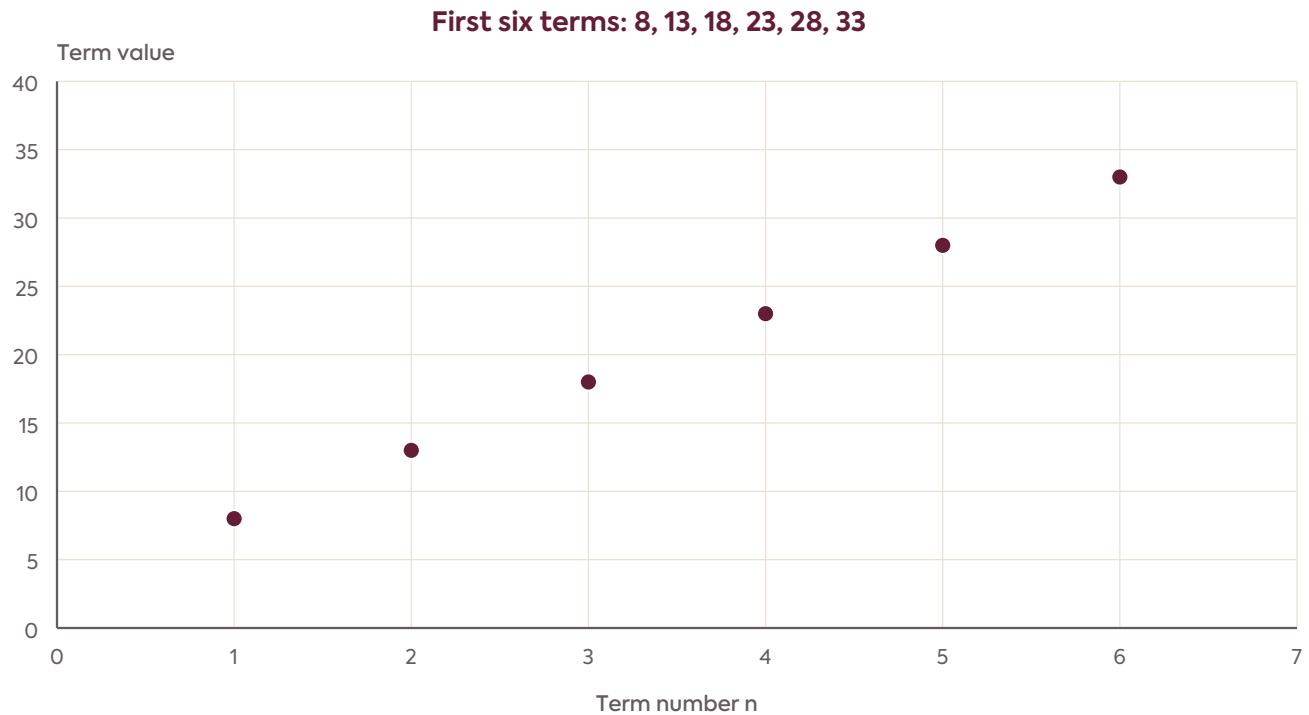
The story limits the rule

If the theater has eight rows, the domain is $n = 1, 2, \dots, 8$. Do not invent row 8.5 or keep extending the physical theater beyond its actual rows.

A zero common difference produces a constant arithmetic sequence, such as 6, 6, 6, Arithmetic means constant difference, not necessarily an increase.

LESSON 39 | CONNECT THE RULE TO A LINE

Dots are the sequence graph



The plotted points are (1, 8), (2, 13), ... , (6, 33). The graph of these six terms contains **only the six dots**. There is no term numbered 2.5, so do not connect the dots as if all inputs were allowed.

The same values lie on a line

$$a_n = 8 + 5(n - 1)$$

Begin with the explicit sequence rule.

$$a_n = 5n + 3$$

Expand and combine constants.

$$y = 5x + 3$$

This continuous line contains the dots, but has a larger domain.

The common difference 5 is the line's rate of change per one index step. This relationship is **not proportional**: the line $y = 5x + 3$ does not pass through the origin. For n starting at 1, an arithmetic rule is proportional to n when $a_1 = d$.

LESSON 39 | SHORT CHECKS

Your turn

Assume every listed sequence continues with the stated arithmetic pattern. Unless stated otherwise, the first term has index $n = 1$.

1. For 3, 7, 11, 15, ... , write recursive and explicit rules. Find term 20.

2. For 18, 15, 12, ... , find d and the index of the first zero term.

3. $a_n = 7 + 2(n - 1)$, $n = 1, 2, \dots$. List the first three terms and find a_{12} .

4. $a_1 = -2$; $a_n = a_{n-1} + 5$ for $n = 2, 3, \dots$. Find terms 2–4 and an explicit rule.

5. Is 7, 14, 21, 28, ... proportional to its term number n ? Give a rule and a reason.

6. Why should the first six terms of a sequence be drawn as six dots? Is $n = 2.5$ an allowed index here?

LESSON 39 | WORKED ANSWERS

Check the start and every jump

1. $d = 4$. Recursive: $a_1 = 3$; $a_n = a_{n-1} + 4$ for $n = 2, 3, \dots$. Explicit: $a_n = 3 + 4(n - 1)$ for $n = 1, 2, \dots$. Term 20 is $3 + 4(19) = 79$.

2. $d = -3$. Set $18 - 3(n - 1) = 0$. Then $n - 1 = 6$, so $n = 7$. The seventh term is zero; the value zero is not its index.

3. At $n = 1, 2, 3$, the values are **7, 9, 11**. $a_{12} = 7 + 2(11) = 29$.

4. Terms 2–4 are **3, 8, 13**. Explicit: $a_n = -2 + 5(n - 1) = 5n - 7$, $n = 1, 2, \dots$. Check $n = 1$ gives -2 .

5. **Yes**. $a_n = 7 + 7(n - 1) = 7n$, $n = 1, 2, \dots$. The ratio a_n/n is always 7. Its dots lie on $y = 7x$, a line through the origin.

6. Only the indices **1, 2, ..., 6** are included in the six-term graph. **2.5 is not an allowed index**. Connecting the dots would add inputs that are not sequence terms.

I can...

- ☐ Separate an index from its term value.
- ☐ Include an initial value and index range in a recursive rule.
- ☐ Explain why an arithmetic sequence is graphed with dots.

Next class: patterns that multiply instead of add. Curriculum focus: A.12C–D; review A.3B and A.2A.

LESSON 40 | MEET THE IDEA

Geometric sequences and multiplicative change

Thursday, March 25, 2027 . 6:00–7:00 PM Central

By the end, I can...

Identify a common ratio, write recursive and explicit geometric rules, and distinguish multiplication from repeated addition.

An **arithmetic** sequence adds a fixed amount. A **geometric** sequence multiplies by a fixed number called the **common ratio** r . We again use $n = 1, 2, 3, \dots$.

Term number n	1	2	3	4
Term value a_n	3	6	12	24

Divide the new term by the previous term

$6 \div 3 = 2$, $12 \div 6 = 2$ and $24 \div 12 = 2$.

The common ratio is $r = 2$. Each step doubles the value.

The differences 3, 6 and 12 are not constant.

Check several consecutive ratios. For this method, the previous term must be nonzero: division by zero is undefined. Our ratio examples use nonzero first terms and nonzero ratios.

Multiplying by a fraction is still multiplying

80, 40, 20, 10, ... has $r = 1/2$. Each term is half the previous term. A geometric sequence does not have to increase.

LESSON 40 | GEOMETRIC RULES

Count the multiplication steps

Recursive rule

$$a_1 = 3; a_n = 2a_{n-1} \text{ for } n = 2, 3, \dots$$

Start with 3, then multiply the previous term by 2.

Explicit rule for 3, 6, 12, 24, ...

$$a_1 = 3 \times 2^0$$

At term 1, there have been zero multiplication steps.

$$a_2 = 3 \times 2^1$$

At term 2, there has been one step.

$$a_n = 3 \times 2^{n-1}$$

For $n = 1, 2, \dots$, there are $n - 1$ steps.

$$a_6 = 3 \times 2^5 = 96$$

Use the fifth power for the sixth term.

General geometric rules

Explicit: $a_n = a_1 r^{n-1}$, $n = 1, 2, \dots$

Recursive: give a_1 , then $a_n = r a_{n-1}$ for $n = 2, 3, \dots$

The first index and first value must be stated.

An exponent counts factors, not an amount to add. For example, 3×2^5 means 3×32 , not $3 \times 2 \times 5$.

LESSON 40 | TWO USEFUL CASES

Fractions shrink; negative ratios alternate

160, 80, 40, 20, ...

$$r = 80/160 = 1/2$$

Each term keeps half of the previous value.

$$a_n = 160(1/2)^{n-1}$$

The first term is $n = 1$.

$$a_7 = 160/64 = 2.5$$

Six halvings occur between terms 1 and 7.

2, -6, 18, -54, ...

$$r = -6/2 = -3$$

18/(-6) and -54/18 are also -3.

$$a_n = 2(-3)^{n-1}$$

Parentheses keep the negative sign inside the base.

$$a_5 = 2(-3)^4 = 162$$

An even power of a negative base is positive.

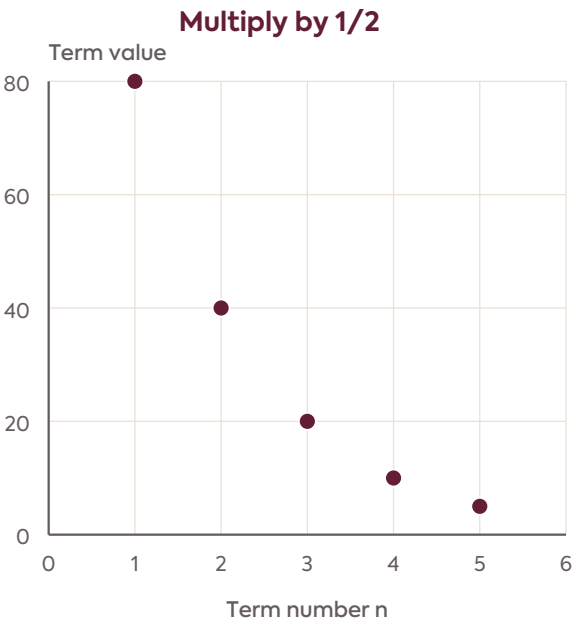
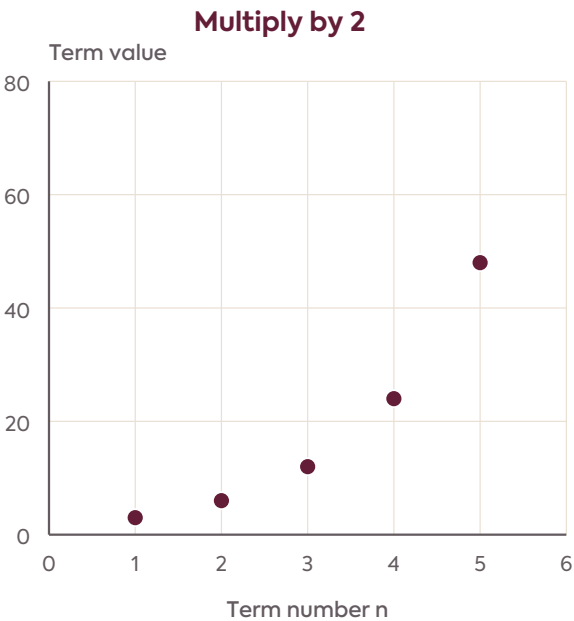
Do not turn a negative ratio into a continuous curve

The rule $2(-3)^{n-1}$ works for integer term numbers. It is **not** a standard real exponential function on every real input: a negative base does not give a real value for every fractional exponent.

For a positive starting value: $r > 1$ increases values; $0 < r < 1$ decreases them toward zero; $r = 1$ keeps them constant. A negative ratio alternates signs.

LESSON 40 | COMPARE THE PATTERNS

Look for equal ratios, not a straight line



Both panels use the same scales. Each dot is a term; no fractional indices are included. The doubling pattern bends upward, while the halving pattern gets closer to zero.

Sequence (n begins at 1)	Test	Explicit rule
3, 6, 9, 12, ...	Difference always 3: arithmetic	$a_n = 3 + 3(n - 1) = 3n$
3, 6, 12, 24, ...	Ratio always 2: geometric	$a_n = 3 \times 2^{n-1}$
2, 4, 7, 11, ...	Neither differences nor ratios stay constant	Neither pattern applies

A number pattern may be neither - or both

Do not force every list into one type. A nonzero constant sequence, such as 5, 5, 5, ... , is both arithmetic ($d = 0$) and geometric ($r = 1$).

LESSON 40 | SHORT CHECKS

Your turn

First index: $n = 1$. Assume each pattern continues as stated. Include the first term and allowed indices when writing a recursive rule.

1. For 5, 15, 45, ... , write both rules and find a_5 .

2. For 96, 48, 24, ... , find r , an explicit rule and a_6 .

3. $a_1 = 4$; $a_n = -2a_{n-1}$ for $n = 2, 3, \dots$. List the first four terms and give an explicit rule.

4. Classify each: (a) 4, 9, 14, 19, ... ; (b) 4, 8, 16, 32, ... ; (c) 1, 2, 4, 7, Explain your tests.

5. A student uses $a_n = 7 \times 3^n$ for 7, 21, 63, ... with first index 1. Correct the rule and explain.

6. Can $2(-3)^{n-1}$ be graphed as a standard real exponential curve for every real n ? Why?

LESSON 40 | WORKED ANSWERS

Check the factor and the exponent

1. $r = 3$. Recursive: $a_1 = 5$; $a_n = 3a_{n-1}$, $n = 2, 3, \dots$. Explicit: $a_n = 5 \times 3^{n-1}$, $n = 1, 2, \dots$. $a_5 = 5 \times 81 = 405$.

2. $r = 1/2$. The rule is $a_n = 96(1/2)^{n-1}$, $n = 1, 2, \dots$. $a_6 = 96/32 = 3$.

3. The terms are 4, -8, 16, -32. Explicit: $a_n = 4(-2)^{n-1}$, $n = 1, 2, \dots$. The signs alternate because the common ratio is negative.

4. (a) **Arithmetic**: differences are 5. (b) **Geometric**: ratios are 2. (c) **Neither**: differences 1, 2, 3 are not constant, and ratios 2, 2, 7/4 are not constant.

5. Use $a_n = 7 \times 3^{n-1}$, $n = 1, 2, \dots$. The first term uses exponent zero: $7 \times 1 = 7$. The proposed rule gives 21 at $n = 1$ and starts one step too far ahead.

6. **No**. The integer-indexed sequence is valid. But, for example, $n = 1.5$ would require the square root of -3, which is not real. Standard real exponential functions use a positive base.

I can...

- ☐ Distinguish a common ratio from a common difference.
- ☐ Use $n - 1$ when the first term has index 1.
- ☐ Handle fractional and negative ratios carefully.

Next class: build exponential growth and decay models. Curriculum focus: A.12C-D; review A.11B.

LESSON 41 | MEET THE IDEA

Writing exponential growth and decay models

Tuesday, March 30, 2027 . 6:00–7:00 PM Central

By the end, I can...

Build an exponential model from an initial value and a repeated percentage change, and explain what each parameter means.

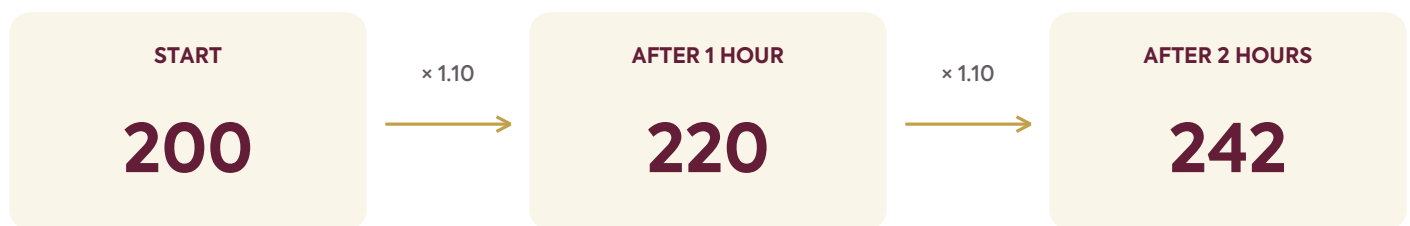
An initial amount times a repeated multiplier

$$f(x) = ab^x$$

a is the output at $x = 0$; **b** is the factor for each increase of 1 in x .

For our nonconstant real exponential models, $b > 0$, $b \neq 1$ and $a \neq 0$.

For positive quantities we use $a > 0$. Growth has $b > 1$. Decay has $0 < b < 1$. The exponent counts equal time periods, not the amount added.



A model starts at 200 and grows 10% per hour.

$$b = 1 + 0.10 = 1.10$$

Keep the original 100%, then add 10%.

$$f(t) = 200(1.10)^t$$

t is hours since the start, with $t = 0$ initially.

$$f(2) = 200(1.10)^2 = 242$$

The second 10% increase is based on 220, not 200.

LESSON 41 | WHAT STAYS AFTER ONE PERIOD?

Turn a percentage into a multiplier

Change per period	Decimal rate r	Multiplier b
Increase by 8%	0.08	$1 + 0.08 = 1.08$
Decrease by 8%	0.08	$1 - 0.08 = 0.92$
Increase by 25%	0.25	1.25
Decrease by 25%	0.25	0.75

A fictional display fades by 20% each minute from intensity 500.

$$b = 1 - 0.20 = 0.80$$

A 20% loss leaves 80%, not 20%.

$$I(t) = 500(0.80)^t$$

t is the number of minutes since fading began.

$$I(3) = 500(0.80)^3 = 256$$

After three minutes, the model predicts 256 intensity units.

Reverse the conversion

For a positive quantity: **$b = 1.06$** means a 6% increase.

$b = 0.94$ means a 6% decrease.

If $b > 1$, growth rate = $b - 1$. If $0 < b < 1$, decay rate = $1 - b$. Multiply the decimal rate by 100 to express it as a percent.

LESSON 41 | USE EQUAL INPUT STEPS

Write the model from a table

Time t (hours)	0	1	2	3
Output	80	100	125	156.25

The table follows an exact exponential rule.

- $a = 80$

$b = 100/80 = 1.25$

$f(t) = 80(1.25)^t$
- Read the output at $t = 0$.

Inputs are one hour apart. Check $125/100$ and $156.25/125$ too.

The model grows by 25% per hour.

What if $t = 0$ is not listed?

- $f(1) = 60; f(2) = 90$

$b = 90/60 = 1.5$

$a = 60/1.5 = 40$

$f(t) = 40(1.5)^t$
- Assume a positive-base exponential model.

The two input values differ by 1.

Step backward once to $t = 0$.

Check $t = 1$ gives 60 and $t = 2$ gives 90.

Real data usually have imperfect ratios; we will fit such data in Class 43. A model assumption matters: a few points alone do not prove a unique rule for all inputs.

LESSON 41 | TIME PERIODS AND SEQUENCE TERMS

Keep the clock and the index clear

A quantity starts at 40 and doubles every 3 hours.

Number of doubling periods = $t/3$

$P(t) = 40 \times 2^{t/3}$

$P(6) = 40 \times 2^2 = 160$

t is measured in hours.

One doubling occurs at $t = 3$, two at $t = 6$.

40×2^6 would incorrectly double every hour.

Meaning	Input convention	Rule
Amount after t hours	$t = 0$ is the start	$f(t) = 40 \times 2^{t/3}$
Readings every 3 hours, starting now	Reading 1 is at $t = 0$	$a_n = 40 \times 2^{n-1}$, $n = 1, 2, \dots$

Choose continuous or discrete inputs from the story

If the process is modeled throughout time, allow real $t \geq 0$.

If only completed hourly rounds count, use $t = 0, 1, 2, \dots$.

A sequence of readings has integer indices even when the underlying process is continuous.

The hourly multiplier in the doubling example is $2^{1/3} \approx 1.260$. Thus $40(2^{1/3})^t$ is equivalent to $40 \times 2^{t/3}$. Units determine which multiplier to use.

LESSON 41 | SHORT CHECKS

Your turn

1. A model begins at 120 and grows 5% per day. Write $f(t)$ and find $f(2)$, where t is days since the start.

2. A signal starts at 800 units and decreases 15% per hour. Write $S(t)$ and find $S(2)$.

3. For $f(t) = 60(0.7)^t$, explain 60 and 0.7. What is the percent decrease per time period?

4. An exact exponential table has $(t, y) = (0, 12), (1, 18), (2, 27)$. Find the rule and percentage growth.

5. A quantity starts at 10 and triples every 2 hours. Write a model using t in hours and find its value at $t = 4$.

6. Readings begin at time 0 for $f(t) = 5(1.2)^t$ and are taken hourly. Give the sequence rule when the first reading is $n = 1$.

LESSON 41 | WORKED ANSWERS

Check the amount retained

1. $f(t) = 120(1.05)^t$, with t in days since the start. $f(2) = 120 \times 1.1025 = \mathbf{132.3}$. The multiplier includes the original 100% plus 5%.
2. A 15% decrease leaves 85%: $S(t) = 800(0.85)^t$. $S(2) = 800 \times 0.7225 = \mathbf{578 \text{ units}}$.
3. **60** is the initial output at $t = 0$. **0.7** means 70% remains after each period, a **30% decrease** because $1 - 0.7 = 0.3$.
4. $a = 12$ and $b = 18/12 = 27/18 = 1.5$. The rule is $f(t) = 12(1.5)^t$; it grows by **50%** per one-unit increase in t .
5. $P(t) = 10 \times 3^{t/2}$. Four hours contains two tripling periods: $P(4) = 10 \times 9 = \mathbf{90}$.
6. Reading n occurs at $t = n - 1$. Therefore $a_n = 5(1.2)^{n-1}$, $n = 1, 2, \dots$. The first reading is 5, not 6.

I can...

- ☐ Translate a percentage increase or decrease into a multiplier.
- ☐ State the units and meaning of the input.
- ☐ Distinguish time 0 from sequence term 1.

Next class: graph these models and describe their domains and ranges. Curriculum focus: A.9B–C; review A.11B and A.12D.

LESSON 42 | MEET THE IDEA

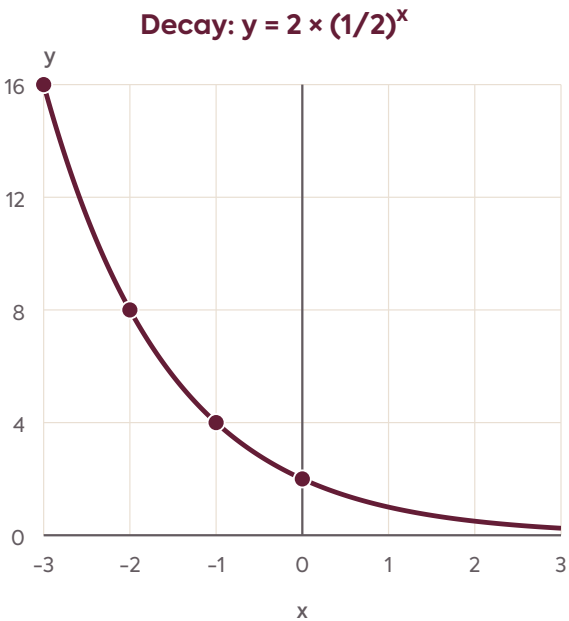
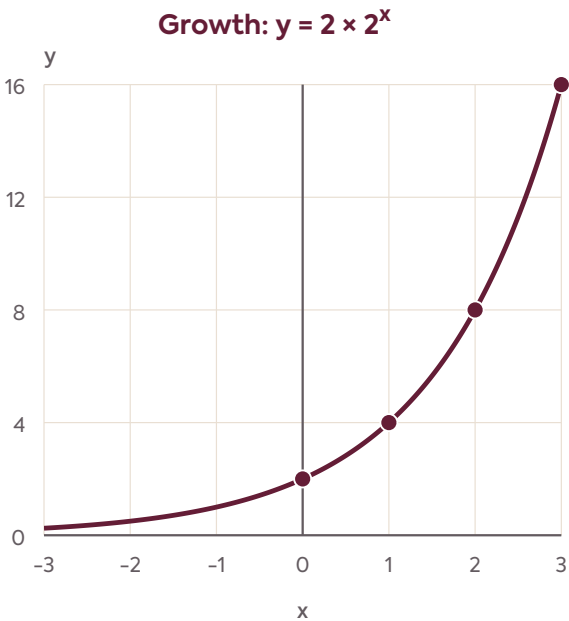
Exponential graphs, domains, ranges and asymptotes

Thursday, April 1, 2027 . 6:00–7:00 PM Central

By the end, I can...

Graph exponential functions, identify intercepts and asymptotes, and describe domains and ranges that match the context.

Begin with a table, then draw a smooth curve if the input is continuous. These two unrestricted functions have positive starting values and the same y-intercept (0, 2).



x	-2	-1	0	1	2
2×2^x	1/2	1	2	4	8
$2 \times (1/2)^x$	8	4	2	1	1/2

Both curves continue beyond the windows. For a positive starting value, $b > 1$ gives an increasing curve; $0 < b < 1$ gives a decreasing curve. The marked dots help draw the curve; they do not limit its domain.

LESSON 42 | DOMAIN, RANGE AND ASYMPTOTE

Approach zero without reaching it

For an unrestricted $f(x) = ab^x$

Assume **$a \neq 0$, $b > 0$ and $b \neq 1$.**

Domain: all real x .

y-intercept: **$(0, a)$** , because $b^0 = 1$.

Horizontal asymptote: **$y = 0$.**

No x-intercept: b^x is positive, so ab^x cannot equal zero.

A **horizontal asymptote** is a horizontal line the curve approaches in an end direction. For $f(x) = 2(1/2)^x$, the outputs approach 0 as x becomes very large, but they remain positive.

x	4	6	8	10
$2(1/2)^x$	0.125	0.03125	0.0078125	0.001953125

The sign of a determines the unrestricted range

If **$a > 0$** , the range is **$y > 0$** , or $(0, \infty)$.

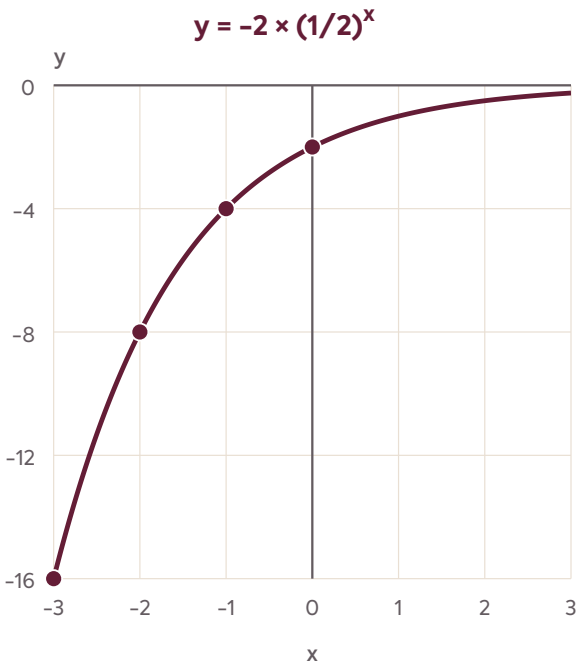
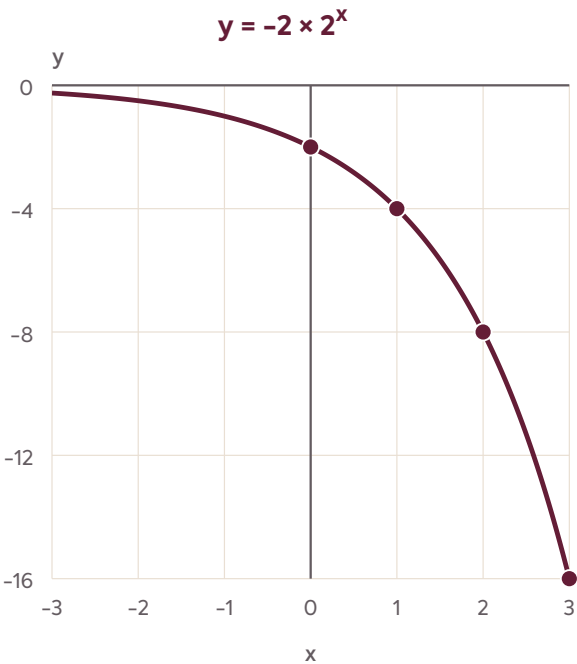
If **$a < 0$** , the range is **$y < 0$** , or $(-\infty, 0)$.

Zero is excluded in both cases. Parentheses mean the endpoint is not included.

A calculator may round a tiny output to 0 or make the curve appear to touch the axis. That display does not make the exact exponential value zero. These rules are for the unshifted form ab^x .

LESSON 42 | KEEP THE BASE POSITIVE

A negative a reflects the curve



Multiplying the whole function by -1 reflects it across the x -axis. The **base is still positive**. Both graphs have y -intercept $(0, -2)$, domain all real numbers, range $y < 0$ and asymptote $y = 0$.

Function	As x increases	What approaches zero?
-2×2^x	Decreases: $-2, -4, -8, \dots$	Outputs approach 0 from below as x heads toward $-\infty$.
$-2 \times (1/2)^x$	Increases: $-2, -1, -1/2, \dots$	Outputs approach 0 from below as x heads toward $+\infty$.

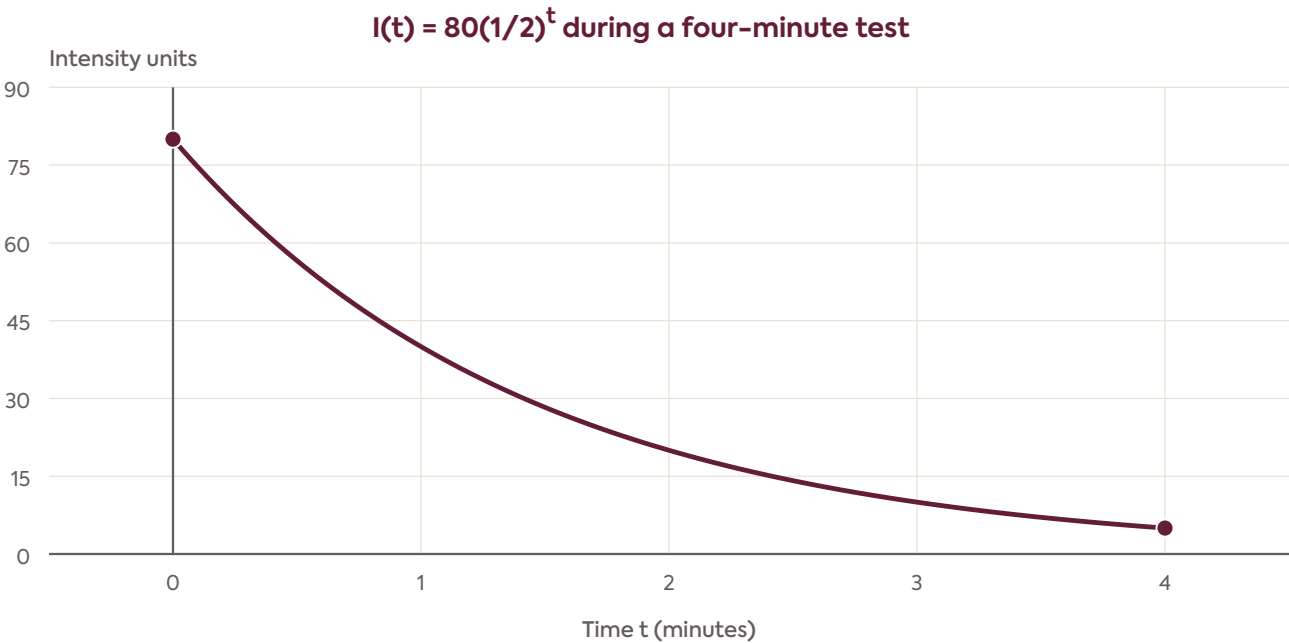
Coefficient sign is not base sign

-2×2^x has positive base 2 and is real for every real x .

$2 \times (-2)^x$ has negative base -2 and is not a standard real exponential function on all real inputs.

LESSON 42 | RESTRICT THE GRAPH AND THE RANGE TOGETHER

The context can shrink the domain



During the test, $0 \leq t \leq 4$. Intensity decreases continuously from 80 to 5, so the model range is $5 \leq I \leq 80$. Filled endpoint dots show that both endpoints are included.

Allowed inputs for the same formula	Corresponding outputs
Every real t	All positive values: $I > 0$
Continuous time $t \geq 0$	$0 < I \leq 80$
Only readings $t = 0, 1, 2, 3, 4$	Exactly $\{80, 40, 20, 10, 5\}$

Do not give the unrestricted range for a restricted story

The four-minute segment never gets below 5. The asymptote $y = 0$ describes the extended function, not a value reached during the test. Discrete readings include only listed outputs, not every value between them.

LESSON 42 | SHORT CHECKS

Your turn

1. For $f(x) = 3 \times 2^x$, find $f(-1)$, $f(0)$, $f(1)$ and $f(2)$. Is it increasing or decreasing?

2. For unrestricted $g(x) = 5(1/3)^x$, state domain, range, y-intercept, horizontal asymptote and any x-intercept.

3. For $h(x) = -4 \times 2^x$, state its range and y-intercept. Does h increase or decrease as x increases?

4. The model $I(t) = 64(1/2)^t$ applies for continuous time $0 \leq t \leq 3$. State its domain and range.

5. For the same model, suppose only $t = 0, 1, 2, 3$ readings are allowed. List the domain and range. Should the dots be connected?

6. A calculator displays 0 for $7(1/2)^{100}$. Is the exact value zero? Explain using the formula.

LESSON 42 | WORKED ANSWERS

Check the allowed inputs first

1. The outputs are **1.5, 3, 6 and 12**. The function is **increasing**: $a = 3 > 0$ and $b = 2 > 1$.
2. Domain: **all real numbers**. Range: **$y > 0$** . y-intercept: **(0, 5)**. Horizontal asymptote: **$y = 0$** . There is **no x-intercept** because a positive exponential output never equals zero.
3. Range: **$y < 0$** . y-intercept: **(0, -4)**. The function **decreases**; for example, $h(0) = -4$, $h(1) = -8$ and $h(2) = -16$. "Increasing base" does not override a negative coefficient.
4. Domain: **$0 \leq t \leq 3$** . $l(0) = 64$ and $l(3) = 8$. The function decreases continuously, so its range is **$8 \leq l \leq 64$** .
5. Domain: **{0, 1, 2, 3}**. Range: **{64, 32, 16, 8}**. Use four separate dots; connecting them would include times and outputs outside the discrete reading set.
6. **No.** $7(1/2)^{100} = 7/2^{100}$ is positive. The calculator display is rounded; the exact function has not reached its asymptote.

Two boundary cases

If $a = 0$, $f(x) = 0$ is the zero function. If $b = 1$, $f(x) = a$ is constant. These are outside the nonconstant exponential cases studied here.

I can...

- ☐ Use coefficient sign to decide an unrestricted range.
- ☐ Distinguish a plotted curve from discrete readings.
- ☐ Restrict both domain and range to the context.

Next class: fit a model to data and review Chapter 7. Curriculum focus: A.9A and A.9D; review A.9B-C.

LESSON 43 | MEET THE IDEA

Exponential regression and the Unit 7 checkpoint

Tuesday, April 6, 2027 . 6:00–7:00 PM Central

By the end, I can...

Use technology to fit an exponential model, interpret its parameters and predictions, and review sequence and exponential skills.

Real measurements rarely have perfectly equal ratios. **Exponential regression** uses technology to fit an exponential curve to paired data. The curve need not pass through every dot.

Time t (hours)	Signal reading (units)
0	100
1	81
2	63
3	52
4	40
5	33

Our modeling question

A fictional signal weakens over five hours. Can an exponential model summarize these positive readings and estimate the signal between observations?

The dataset is a teaching example, not measurements from a real device.

The ratios $81/100 = 0.81$ and $63/81 \approx 0.778$ are close but not equal. They suggest considering decay; they do not make any one ratio an exact answer.

LESSON 43 | GUIDED TECHNOLOGY ACTIVITY

Perform the fit yourself

Use the teacher-approved graphing tool. The steps below use [Desmos Graphing Calculator](#). Start a fresh graph so a and b are not already assigned values.

Step	What to do
1 . Enter the data	Add a table. Enter times 0, 1, 2, 3, 4, 5 in x_1 and readings 100, 81, 63, 52, 40, 33 in y_1 . Keep each pair in the same row.
2 . Ask for regression	On a new expression line type: $y_1 \sim a b^{(x_1)}$ Underscores make subscripts. The tilde ~ asks for a fit; an equals sign does not perform regression.
3 . Match the worked setting	Leave Log Mode off . This fits the original signal readings by minimizing squared vertical errors.
4 . Record the parameters	You should get about $a = 100.291$ and $b = 0.798844$. Keep the unrounded fitted values in the calculator.
5 . Inspect and predict	Enter $f(x) = a b^x$. Zoom to see the dots and curve together. Evaluate $f(2.5)$, then compare $f(3)$ with the observed reading.

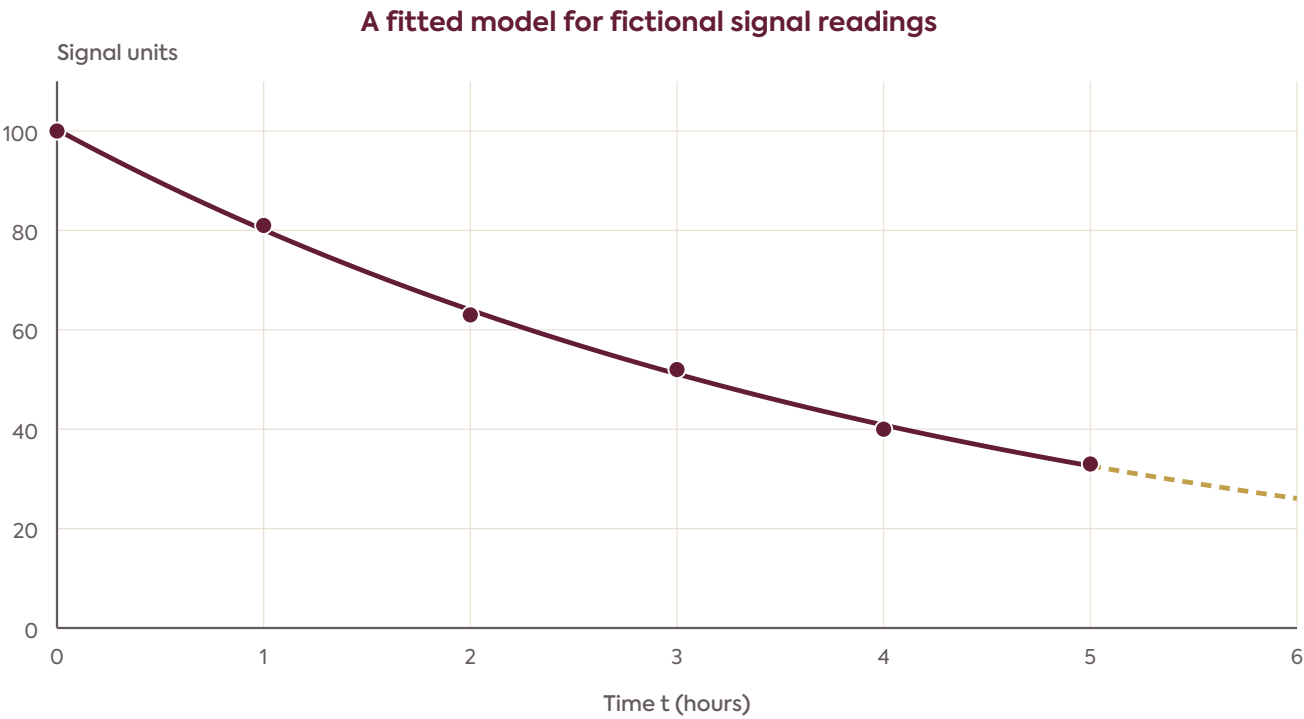
A different regression setting can give a different fit

Log Mode fits transformed data and may produce different coefficients. Other calculators can also use that approach. For these worked answers, use the original-data fit above. Do not treat a small method difference as an arithmetic mistake.

Official help: [Regressions](#) and [Log Mode](#). If results differ substantially, check every table entry and the model form first.

LESSON 43 | PARAMETERS HAVE MEANING

Explain the fitted curve



Our fitted model

$f(t) \approx 100.291(0.798844)^t$

t is hours since the first reading; f(t) is the predicted signal in units.
The displayed coefficients are rounded. Use the unrounded calculator parameters for predictions.

Parameter	Interpretation
$a \approx 100.291$	Predicted starting signal at $t = 0$. It is close to, but not exactly, the observed 100.
$b \approx 0.798844$	About 79.88% is retained each hour: a decrease of about 20.12% per hour.

The solid curve covers observed times 0–5 hours. The dashed part extends beyond the data. A close fit describes these readings; it does not guarantee future behavior or prove a cause.

LESSON 43 | ESTIMATE, COMPARE AND EXPLAIN

Predict carefully, then review the pattern

Input	Model estimate	Meaning
$t = 2.5$	$f(2.5) \approx 57.20$ units	Interpolation: between observed times.
$t = 3$	$f(3) \approx 51.13$ units	Observed value is 52; the fitted curve is not exact.
$t = 10$	$f(10) \approx 10.61$ units	Extrapolation: well beyond the 0–5 hour data interval.

A **residual** is observed minus predicted. At $t = 3$ it is $52 - 51.12656... \approx \mathbf{0.87 \text{ units}}$. A positive residual means the dot is above the fitted curve.

Turn the fitted model into an hourly sequence

If reading 1 corresponds to time 0, the predicted readings follow

$s_n = a b^{n-1}$, $n = 1, 2, \dots$, using the fitted a and b .

Equivalently, $s_1 = a$ and $s_n = b s_{n-1}$ for $n = 2, 3, \dots$.

Measured readings need not form an exact geometric sequence.

Today’s 60-minute lesson

5 minutes: retrieval . 15: model demonstration . 20: guided fitting and interpretation . 15: separate Unit 7 checkpoint . 5: next steps.

Use the following study checks for revision before or after class. Your teacher provides the graded checkpoint separately. It checks rules and indexing, percentage change, graph features, and model interpretation.

LESSON 43 | STUDY CHECKS BEFORE THE CHECKPOINT

Your turn

1. Fit the supplied signal data with Log Mode off. Record a and b to at least six significant figures, then interpret the hourly percent decrease.

2. Use the unrounded fit to estimate $f(2.5)$. Is this interpolation or extrapolation? Give units.

3. Estimate $f(3)$, then compute the residual for the observed reading 52. Explain its sign.

4. Estimate $f(10)$. Explain one reason that this estimate is less secure than $f(2.5)$.

5. Compare (a) 10, 15, 20, 25, ... and (b) 10, 15, 22.5, 33.75, For each, give both rules with first index $n = 1$.

6. For the unshifted fitted curve on $t \geq 0$, state the model range and asymptote. Would a very small prediction prove the actual signal can never be zero?

LESSON 43 | WORKED ANSWERS

Connect the method to the meaning

1. The original-data fit gives $a \approx 100.290847$, $b \approx 0.798843599$. The hourly decrease is $(1 - b) \times 100\% \approx 20.12\%$. It retains about 79.88%, not 20.12%.
2. $f(2.5) \approx 57.20$ signal units. This is **interpolation** because 2.5 lies between observed input values within 0–5 hours.
3. $f(3) \approx 51.13$ units. Using the unrounded prediction, residual = $52 - 51.12656... \approx +0.87$ units. The observed dot lies above the curve.
4. $f(10) \approx 10.61$ units. This is **extrapolation**; no readings were supplied after hour 5. The process or conditions may change, so the fitted pattern may not continue.
5. (a) Arithmetic, $d = 5$. Explicit: $a_n = 10 + 5(n - 1)$, $n = 1, 2, \dots$. Recursive: $a_1 = 10$; $a_n = a_{n-1} + 5$, $n = 2, 3, \dots$.
(b) Geometric, $r = 1.5$. Explicit: $b_n = 10(1.5)^{n-1}$, $n = 1, 2, \dots$. Recursive: $b_1 = 10$; $b_n = 1.5b_{n-1}$, $n = 2, 3, \dots$.
6. For $t \geq 0$, the model range is $0 < f(t) \leq a$, where $a \approx 100.290847$, and the horizontal asymptote is $y = 0$. This mathematical model never outputs zero; that does **not** prove the real process can never reach zero or stop following the model.

Choose your next practice

Class 39: differences and indexing. Class 40: ratios and sequence rules.

Class 41: multipliers and time units. Class 42: domains, ranges and asymptotes. Class 43: technology, parameters and prediction limits.

Chapter 8 begins Thursday, April 8, 2027: choosing linear, quadratic or exponential models. Curriculum focus: A.9E, with A.9A–D and A.12C–D revisited.

KEEP THIS PAGE FOR REVISION

Chapter 7 quick reference

Idea	Remember
Arithmetic rules	$a_n = a_1 + (n - 1)d$, $n = 1, 2, \dots$. Recursive: state a_1 ; $a_n = a_{n-1} + d$, $n = 2, 3, \dots$.
Geometric rules	$a_n = a_1 r^{n-1}$, $n = 1, 2, \dots$. Recursive: state a_1 ; $a_n = r a_{n-1}$, $n = 2, 3, \dots$.
Index versus time	If reading 1 is at time 0, its time is $t = n - 1$. Sequence graphs use integer-indexed dots.
Exponential model	$f(t) = ab^t$: a is $f(0)$; b is the factor per one time unit. Standard nonconstant case: $a \neq 0$, $b > 0$, $b \neq 1$.
Percentage change	For a positive quantity: increase by rate r gives $b = 1 + r$; decrease by rate r gives $b = 1 - r$. Write r as a decimal.
Graph features	For unrestricted ab^x : domain all real numbers; y-intercept $(0, a)$; asymptote $y = 0$; no x-intercept.
Range and context	Unrestricted: $y > 0$ if $a > 0$; $y < 0$ if $a < 0$. Recalculate the range when the input domain is restricted.
Exponential regression	Fit paired data with technology; report units and parameter meanings. For Class 43, keep Log Mode off and use unrounded parameters.
Prediction limits	Interpolation stays within observed inputs. Extrapolation goes outside them. A good fit does not guarantee future behavior.

STUDY GUIDANCE AND CURRICULUM REFERENCES

Make these notes work for you

A small routine for each lesson

Before: recall a difference, ratio or exponent rule.

During: explain why each worked step follows.

After: try the study checks with answers covered, then complete your assigned digital homework.

Study notes and graded assessments

These PDFs include their own revision answers. The Class 40 quiz and Class 43 graded checkpoint are separate teacher-provided materials. Their unpublished questions and keys are not included.

Graphs and modeling

Sequence graphs show only allowed integer indices. Continuous curves are drawn from their stated formulas with labeled scales. An unrestricted curve continues beyond the window. In the regression graph, solid means observed input interval; dashed means extrapolation.

All stories and datasets are fictional teaching examples. Class 43 requires students to perform a technology fit. Its worked results use squared errors on the original outputs; other settings can give different coefficients. A model is an approximation, not a promise.

Curriculum alignment

Standard Algebra I, §111.39(c): A.9A–E and A.12C–D, revisiting A.2A, A.3B and A.11B. Advanced Grade 8 Algebra I, §111.31(d): corresponding (9)(A)–(E), (12)(C)–(D) and review skills.

Original Cambridge Veritas supplementary notes; not an official TEA publication. Dates and lesson order follow our course calendar, not state-required pacing.

Official references

Texas Education Agency: [Standard Algebra I, §111.39, mathematics TEKS](#).

Texas Education Agency: [Advanced Grade 8 Algebra I, §111.31, adopted 2025](#).

Desmos: [Official regression instructions](#).

Desmos: [Log Mode and different regression settings](#).

Standards and software guidance checked September 9, 2026. Chapter 7 edition: 1.0.