

## LESSON 36 | MEET THE IDEA

# The quadratic formula and real-root decisions

Thursday, March 4, 2027 . 6:00–7:00 PM Central

## By the end, I can...

Use the quadratic formula with correct signs and parentheses, identify the number of real roots and write real linear factors when possible.

First write  $ax^2 + bx + c = 0$ , with  $a \neq 0$ . Completing the square on this general equation produces the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The **whole numerator** is divided by  $2a$ . The  $\pm$  gives a plus case and a minus case. A missing  $x$ -term means  $b = 0$ ; a missing constant means  $c = 0$ .

**Solve  $2x^2 - 3x - 2 = 0$**

$$a = 2, b = -3, c = -2$$

$$b^2 - 4ac = 9 + 16 = 25$$

$$x = (3 \pm 5)/4$$

$$x = 2 \text{ or } x = -1/2$$

Include the signs of the coefficients.

$$(-3)^2 - 4(2)(-2) = 25.$$

$$-b = 3 \text{ and } 2a = 4.$$

Compute  $(3 + 5)/4$  and  $(3 - 5)/4$  separately.

## LESSON 36 | AN IRRATIONAL-ROOT EXAMPLE

# Keep radicals exact until the end

Solve  $2x^2 - 4x - 1 = 0$

$$a = 2, b = -4, c = -1$$

$$D = (-4)^2 - 4(2)(-1) \\ = 24$$

$$x = (4 \pm \sqrt{(24)})/4$$

$$x = (4 \pm 2\sqrt{(6)})/4$$

$$x = (2 \pm \sqrt{(6)})/2$$

The equation is already in standard form.

The expression under the radical is called the discriminant,  $D$ .

Substitute  $-b = 4$  and  $2a = 4$ .

Simplify  $\sqrt{(24)}$  as  $2\sqrt{(6)}$ .

Divide EVERY term of the numerator and denominator by 2.

These are approximately **2.22** and **-0.22**. Keep the radical forms for exact work.

## Check without messy decimal arithmetic

The roots can also be written  $x = 1 \pm \sqrt{(6)}/2$ .

Then  $(x - 1)^2 = 6/4 = 3/2$ .

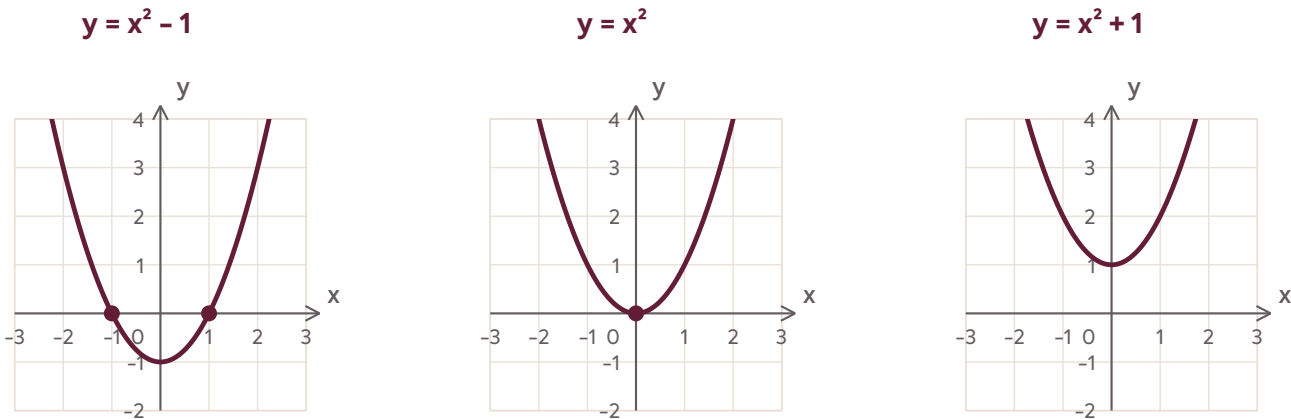
The original expression is  $2(x - 1)^2 - 3$ , which becomes  $2(3/2) - 3 = 0$ .

An equation that does not factor neatly over the integers may still have two real solutions. The formula finds them without guessing an integer factor pair.

LESSON 36 | TWO, ONE OR NO REAL ROOTS

# The discriminant tells you what to expect

For  $ax^2 + bx + c = 0$ , calculate  $D = b^2 - 4ac$ . Its sign tells whether the square-root part of the formula is positive, zero or not real.



| Discriminant | Distinct real roots | Graph of $y = ax^2 + bx + c$      |
|--------------|---------------------|-----------------------------------|
| $D > 0$      | Two                 | Crosses the x-axis twice.         |
| $D = 0$      | One, repeated       | Touches the x-axis at the vertex. |
| $D < 0$      | None                | Does not meet the x-axis.         |

In the graphs above, the discriminants are 4, 0 and -4, respectively. The curves continue outside the windows.

**Stay in the real-number scope**

A negative discriminant means **no real solutions**. Do not write  $\sqrt{(-4)} = -2$ . A real square cannot be negative. Complex-number calculations are outside this lesson.

## LESSON 36 | FINISH THE LINK FROM CHAPTER 5

# Real roots give real linear factors

If a quadratic has real roots  $r_1$  and  $r_2$ , its expression is  $a(x - r_1)(x - r_2)$ . Keep the original leading coefficient  $a$ .

**Factor  $x^2 - 2x - 1$  over the real numbers**

$$D = (-2)^2 - 4(1)(-1) \\ = 8$$

No integer factor pair is needed. Use the formula.

$$x = (2 \pm \sqrt{8})/2 \\ = 1 \pm \sqrt{2}$$

These are the two roots.

$$[x - (1 + \sqrt{2})] \\ \times [x - (1 - \sqrt{2})]$$

Use one linear factor for each root. Here  $a = 1$ .

**Verify with a difference of squares**

$$[(x - 1) - \sqrt{2}][(x - 1) + \sqrt{2}] \\ = (x - 1)^2 - 2 \\ = x^2 - 2x - 1.$$

If the original expression were  $3x^2 - 6x - 3$ , the same roots would apply, but the factorization would need **3 times** the product above. Roots alone do not supply the leading coefficient.

When  $D = 0$ , the form is  $a(x - r)^2$ . When  $D < 0$ , there are no real linear factors; this does not prevent taking out a common constant factor.

## LESSON 36 | SHORT CHECKS

# Your turn

Use the quadratic formula for Questions 1–3, even if another method also works. Give exact answers.

1.  $x^2 - 3x - 4 = 0$

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2.  $2x^2 + 4x - 1 = 0$

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3.  $x^2 - 6x + 9 = 0$

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4. Use the discriminant to classify the real roots of  $x^2 + 2x + 5 = 0$ .

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5. Find the roots of  $x^2 - 4x + 1 = 0$ , then factor  $x^2 - 4x + 1$  over the real numbers.

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6. For  $2x^2 - 3x - 2 = 0$ , a student gets  $x = (3 \pm 5)/2$ . Identify the error and give the corrected roots.

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## LESSON 36 | WORKED ANSWERS

# Check the signs and the denominator

1.  $a = 1$ ,  $b = -3$ ,  $c = -4$ ;  $D = 9 + 16 = 25$ .

$x = (3 \pm 5)/2$ , so  $x = 4$  or  $-1$ . Checks:  $16 - 12 - 4 = 0$  and  $1 + 3 - 4 = 0$ .

2.  $a = 2$ ,  $b = 4$ ,  $c = -1$ ;  $D = 16 + 8 = 24$ .

$x = (-4 \pm 2\sqrt{(6)})/4 = (-2 \pm \sqrt{(6)})/2$ .

Check:  $2x^2 + 4x - 1 = 2(x + 1)^2 - 3$ ; each root makes  $(x + 1)^2 = 3/2$ .

3.  $D = 36 - 36 = 0$ .  $x = (6 \pm 0)/2 = 3$ , one repeated real root. Check:  $9 - 18 + 9 = 0$ .

4.  $D = 4 - 20 = -16$ , so there are **no real roots**.

5.  $D = 16 - 4 = 12$ ;  $x = (4 \pm 2\sqrt{(3)})/2 = 2 \pm \sqrt{(3)}$ .

Factors:  $[x - (2 + \sqrt{(3)})][x - (2 - \sqrt{(3)})]$ .

Check:  $(x - 2)^2 - 3 = x^2 - 4x + 1$ .

6. The denominator must be  $2a = 4$ , not 2.

$x = (3 \pm 5)/4$  gives  $x = 2$  or  $-1/2$ .

## I can...

- ☐ Identify  $a$ ,  $b$  and  $c$  with their signs.
- ☐ Use the discriminant to classify real-root cases.
- ☐ Keep the scale factor when constructing real linear factors.

Next class: fit and interpret a quadratic model from data. Curriculum focus: A.8A, A.10E.