

LESSON 12 | MEET THE IDEA

Transformations of the linear parent function

Thursday, November 19, 2026 . 6:00–7:00 PM Central

By the end, I can...

Predict and explain the effects of $af(x)$, $f(x) + d$, $f(x - c)$ and $f(bx)$ on the parent function $f(x) = x$.

The **parent function** is the starting rule. For this lesson it is **$f(x) = x$** : its output equals its input, its slope is 1, and it passes through the origin.

Change	What is being changed?	Example when $f(x) = x$
$af(x)$	Multiply the output by a .	$2f(x) = 2x$
$f(x) + d$	Add d to the output.	$f(x) + 3 = x + 3$
$f(x - c)$	Replace the input by $x - c$.	$f(x - 3) = x - 3$
$f(bx)$	Multiply the input by b before applying f .	$f(2x) = 2x$

Inside or outside?

An outside operation acts on the **output**. An inside operation changes the **input** sent into f . Write the new rule first; then check a point.

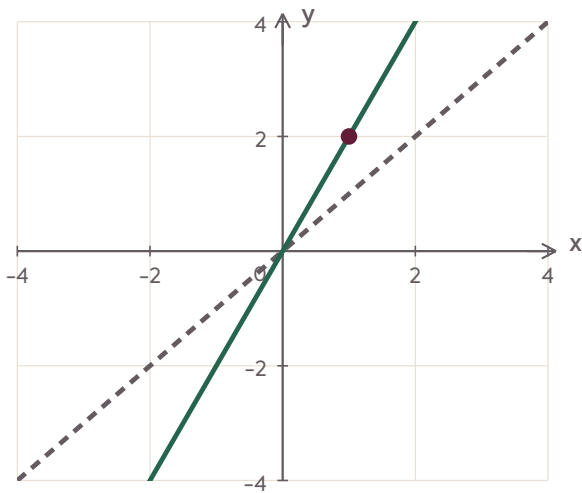
The letters a , b , c and d in this lesson are transformation parameters. The b inside $f(bx)$ is not the y -intercept symbol from $y = mx + b$.

LESSON 12 | VERTICAL CHANGES

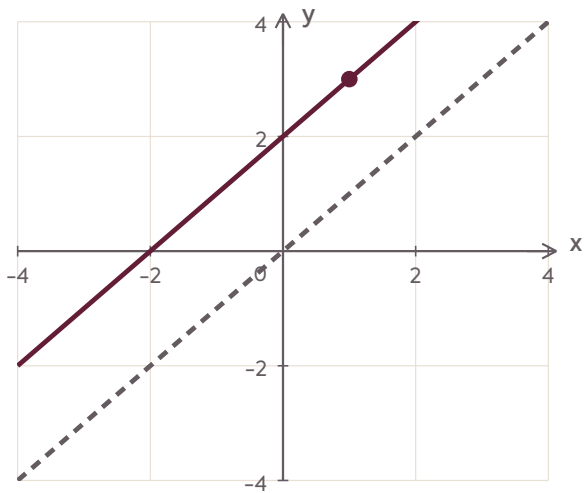
Changes to the output

$af(x)$: multiply every y -coordinate by a . If $|a| > 1$, stretch vertically; if $0 < |a| < 1$, compress vertically. If $a < 0$, also reflect across the x -axis. Here $|a|$ means the size of a without its sign.

Vertical stretch: $2f(x) = 2x$



Shift up: $f(x) + 2 = x + 2$



In both graphs, the dashed line is $f(x) = x$. For $2f(x)$, the parent point $(1, 1)$ becomes **$(1, 2)$** . For $f(x) + 2$, it becomes **$(1, 3)$** .

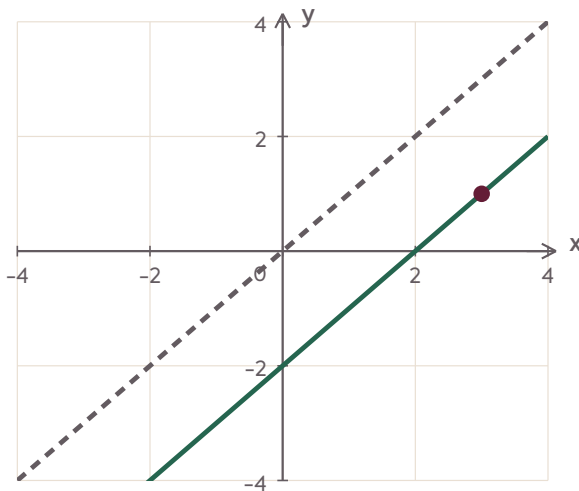
New rule	Effect on the parent graph
$(1/2)f(x) = (1/2)x$	Vertical compression by factor $1/2$.
$-2f(x) = -2x$	Vertical stretch by 2 and reflection across the x -axis.
$f(x) - 3 = x - 3$	Shift down 3 units; slope remains 1.

LESSON 12 | HORIZONTAL CHANGES

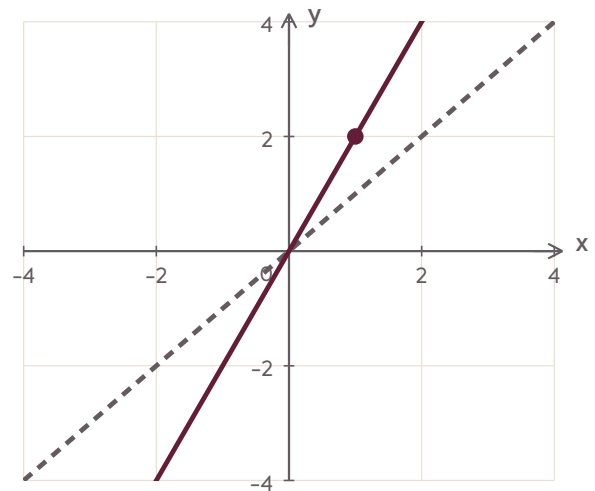
Changes inside the function

$f(x - c)$: shift right c units when $c > 0$; shift left $|c|$ units when $c < 0$. The minus sign inside is why $f(x - 2)$ moves the parent graph **right** by 2.

Shift right: $f(x - 2) = x - 2$



Input scale: $f(2x) = 2x$



For $f(x - 2)$, the parent point (1, 1) moves to **(3, 1)**. For $f(2x)$, the parent point (2, 2) moves to **(1, 2)**: the new input only needs to be half as large.

The factor inside works reciprocally

For $f(bx)$, with $b \neq 0$, each point $(u, f(u))$ moves to **$(u/b, f(u))$** . The horizontal scale factor is $1/|b|$. If b is negative, also reflect across the y-axis.

So $f((1/2)x)$ stretches horizontally by 2, while $f(-2x)$ compresses horizontally by $1/2$ and reflects across the y-axis.

LESSON 12 | WHAT IS SPECIAL ABOUT $F(X) = X$?

Different moves can give the same line

Two descriptions	Both simplify to...	Why they agree here
$2f(x)$ and $f(2x)$	$2x$	The parent simply returns its input.
$f(x) - 3$ and $f(x - 3)$	$x - 3$	A downward shift and a rightward shift produce the same line.
$-f(x)$ and $f(-x)$	$-x$	Reflecting this parent across either axis produces the same line.

Example | Track two different parent points

$2f(x)$: (1, 1) becomes (1, 2)	Multiply the output by 2; the input is unchanged.
$f(2x)$: (2, 2) becomes (1, 2)	Divide the input coordinate by 2; the output is unchanged.

The point moves differ, but the resulting set of points is the same line $y = 2x$.

Do not invent a visual difference

If two rules simplify to the same equation, their graphs coincide. Explain the operation accurately, then say that the resulting line is the same. This coincidence depends on the parent function.

Zero parameters: $0f(x)$ is $y = 0$. Also $f(0x) = f(0) = 0$ for this parent. Do not describe $b = 0$ as scaling by $1/b$; that division is undefined.

LESSON 12 | SHORT CHECKS

Your turn

Use $f(x) = x$ throughout. State the new rule as well as the described transformation.

1. Write $3f(x)$. Where does the parent point $(2, 2)$ move?

2. Write $f(x) - 4$. What happens to the slope and y-intercept?

3. Write $f(x + 2)$. Is the described horizontal shift left or right?

4. Write $f(2x)$. Where does the parent point $(4, 4)$ move? Give the horizontal scale factor.

5. Write $-(1/2)f(x)$. Describe both effects on the output.

6. Do $f(x - 3)$ and $f(x) - 3$ have different graphs? Explain.

One point can reveal the rule

For $g(x) = f(x - 2)$, try $x = 2$: $g(2) = f(0) = 0$. This checks that the old origin has moved to $(2, 0)$.

LESSON 12 | WORKED ANSWERS AND RECAP

Check your thinking

1. $y = 3x$, a vertical stretch by factor 3. The parent point (2, 2) moves to **(2, 6)**.
2. $y = x - 4$, a shift down 4. The slope stays 1; the y-intercept becomes **(0, -4)**.
3. $y = x + 2$. Since $x + 2 = x - (-2)$, the horizontal description is a shift **left 2**.
4. $y = 2x$. The parent point (4, 4) moves to **(2, 4)**. The horizontal scale factor is $\frac{1}{2}$, a compression.
5. $y = -(1/2)x$. Compress vertically by $\frac{1}{2}$ and reflect across the **x-axis**.
6. **No**. Both rules simplify to $y = x - 3$. One describes a rightward shift of this parent; the other describes a downward shift. The final line is identical.

I can...

- ☐ Distinguish input changes from output changes.
- ☐ Use reciprocal horizontal scale factors.
- ☐ Recognize when two transformations give the same line.

Next class: explore scatterplots and fit a line to data. Curriculum focus: A.3E.

The next scheduled class is Tuesday, December 1, after the November break.