

LESSON 22 | MEET THE IDEA

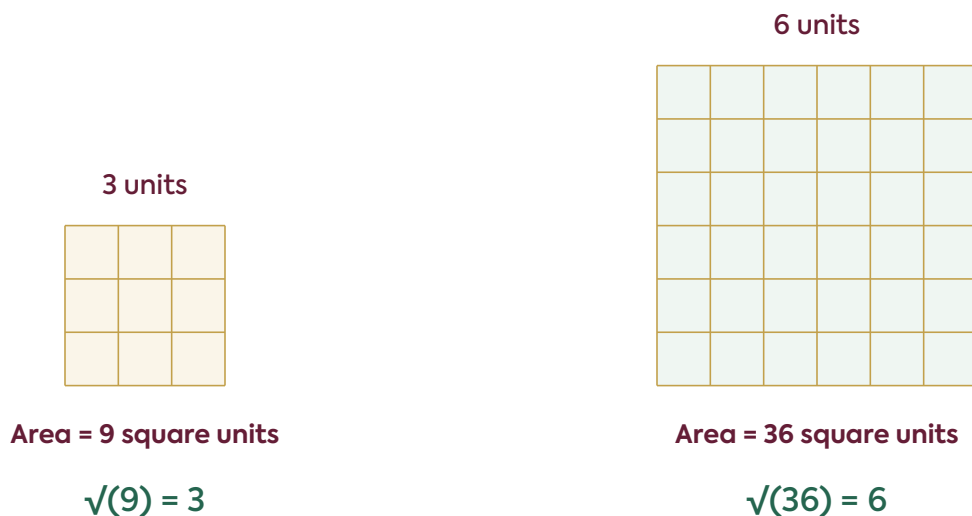
Square roots and rational exponents

Thursday, January 14, 2027 . 6:00–7:00 PM Central

By the end, I can...

Simplify numerical square roots and expressions with rational exponents, using real-number and nonzero restrictions correctly.

Squaring a side length gives a square's area. Taking a **square root** reverses that step. The symbol $\sqrt{}$ means the **principal square root**: the nonnegative one.



Each small square is 1 square unit.

A square root and an equation are different

$\sqrt{(9)} = 3$: the radical symbol names one nonnegative value.

$x^2 = 9$: the equation has two solutions, $x = 3$ and $x = -3$.

Both numbers square to 9, but $\sqrt{(9)}$ does not mean ± 3 .

We write parentheses after $\sqrt{}$ to make clear which expression is inside the root. Over the real numbers, a square root needs a nonnegative radicand: $\sqrt{(0)} = 0$, while $\sqrt{(-9)}$ is not a real number.

LESSON 22 | EXACT RADICAL FORM

Look for a perfect-square factor

Simplify $\sqrt{72}$

$$\sqrt{72} = \sqrt{36 \times 2}$$

Choose the largest perfect-square factor: 36.

$$\sqrt{36} \times \sqrt{2}$$

A square root of a product can split this way when both factors are nonnegative.

$$6\sqrt{2}$$

Since $\sqrt{36} = 6$, take 6 outside the root.

$6\sqrt{2}$ is exact. Its decimal approximation is about 8.49; use $\approx$ for a rounded value.

Combine roots only after simplifying

$$2\sqrt{8} + \sqrt{18}$$

Start with a sum of radicals.

$$2(2\sqrt{2}) + 3\sqrt{2}$$

Use $8 = 4 \times 2$ and $18 = 9 \times 2$.

$$4\sqrt{2} + 3\sqrt{2} = 7\sqrt{2}$$

The radical parts match, so add their coefficients.

Multiplication and division of numerical roots

$$\sqrt{12} \times \sqrt{3} = \sqrt{36} = 6.$$

$$\sqrt{48} / \sqrt{3} = \sqrt{16} = 4.$$

These real-root product rules require nonnegative radicands; a radicand in a denominator must be positive.

LESSON 22 | READ THE NUMERATOR AND DENOMINATOR

A fractional exponent names a root

A **rational exponent** is an exponent written as a fraction. For a **positive base a**, with positive integer n, $a^{1/n}$ means the nth root of a.

Expression	Read it as	Example
$a^{1/2}$	Square root of a	$25^{1/2} = 5$
$a^{1/3}$	Cube root of a	$27^{1/3} = 3$
$a^{m/n}$	Take the nth root, then raise to m.	$27^{2/3} = 3^2 = 9$

Example 1 | $16^{3/4}$

$16^{1/4} = 2$

$(16^{1/4})^3 = 2^3$

$16^{3/4} = 8$

The denominator 4 names a fourth root: $2^4 = 16$.

The numerator 3 tells you the power.

Root first keeps the numbers manageable.

Example 2 | $16^{-1/2}$

$1 / 16^{1/2}$

$1 / 4$

The negative sign in the exponent means a reciprocal.

Take the square root of 16.

LESSON 22 | ALGEBRAIC POWERS AND RESTRICTIONS

The same laws, with safe bases

For the rational-exponent laws on this page, assume $x > 0$ and $a > 0$. This keeps all the roots and quotients real and defined.

Multiply, divide and raise a power

$x^{1/2} \times x^{3/2} = x^2$

$a^{5/3} / a^{2/3} = a$

$(x^{3/2})^2 = x^3$

Add exponents: $1/2 + 3/2 = 2$.

Subtract: $5/3 - 2/3 = 1$.

Multiply: $(3/2) \times 2 = 3$.

Question	What matters
$\sqrt{((-5)^2)}$	First square -5 to get 25 ; its principal square root is 5 , not -5 .
$\sqrt{(-16)}$	No real value: a real square is never negative.
Cube root of -8	It is -2 , because $(-2)^3 = -8$. Odd roots can be negative.
$0^{-1/2}$	Undefined: it would require $1/\sqrt{0}$, which divides by zero.

A root does not distribute over a sum

$\sqrt{9 + 16} = \sqrt{25} = 5$.

But $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$.

The product rule for roots is not a rule for addition.

LESSON 22 | SHORT CHECKS

Your turn

Give exact radical forms unless a decimal is requested. In Question 6, assume $x > 0$.

1. Simplify $\sqrt{50}$. Then approximate it to two decimal places.

2. Simplify $3\sqrt{12} - \sqrt{27}$.

3. Simplify $\sqrt{20} \times \sqrt{5}$.

4. Evaluate $81^{3/4}$ and $25^{-1/2}$.

5. Find $\sqrt{49}$, then solve $t^2 = 49$. Explain the difference.

6. Simplify $(x^{5/2} \times x^{1/2}) / x$. Keep the stated restriction.

7. Is $\sqrt{16 + 9}$ equal to $\sqrt{16} + \sqrt{9}$? Check both sides.

LESSON 22 | WORKED ANSWERS

Check your roots and powers

1. $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$, which is approximately **7.07**. The radical form is exact; the decimal is rounded.
2. $3\sqrt{12} - \sqrt{27} = 3(2\sqrt{3}) - 3\sqrt{3} = 6\sqrt{3} - 3\sqrt{3} = 3\sqrt{3}$.
3. $\sqrt{20} \times \sqrt{5} = \sqrt{100} = 10$. Both radicands are nonnegative.
4. $81^{3/4} = (81^{1/4})^3 = 3^3 = 27$. Also, $25^{-1/2} = 1/\sqrt{25} = 1/5$.
5. $\sqrt{49} = 7$. The equation $t^2 = 49$ has **$t = 7$ or $t = -7$** . The symbol $\sqrt{}$ selects the principal, nonnegative root; the equation asks for every value whose square is 49.
6. $x^{5/2+1/2-1} = x^{3-1} = x^2$, with **$x > 0$** as specified.
7. **No.** $\sqrt{25} = 5$, while $\sqrt{16} + \sqrt{9} = 4 + 3 = 7$.

Exact or approximate?

Use $=$ between exactly equal forms, such as $\sqrt{50} = 5\sqrt{2}$. Use $\approx$ for rounded forms, such as $\sqrt{50} \approx 7.07$.

I can...

- ☐ Extract perfect-square factors and combine matching radicals.
- ☐ Read fractional exponents and use the exponent laws.
- ☐ Distinguish a principal root from the solutions of an equation.

Next class: organize, add and subtract polynomials. Curriculum focus: A.11A, A.11B.