

LESSON 23 | MEET THE IDEA

Adding, subtracting and organizing polynomials

Tuesday, January 19, 2027 . 6:00–7:00 PM Central

By the end, I can...

Organize polynomial terms by degree, combine like terms and rewrite linear and quadratic expressions without changing their value.

A **polynomial in x** is a sum of terms such as $4x^2$, $-3x$ and 7 . Its variable exponents are nonnegative integers. A term's **coefficient** is its numerical factor.

Polynomial	Degree	Number of terms
$4x^2 - 3x + 7$	2: quadratic	3: trinomial
$5x - 8$	1: linear	2: binomial
$-2x^2$	2: quadratic	1: monomial
6	0: constant	1: monomial

Degree is not the number of terms

For a nonzero polynomial in one variable, the degree is the highest exponent with a nonzero coefficient. The expression $7x^2 - 9$ has degree 2 and two terms; x^2 has degree 2 and only one term.

Write polynomials in **descending powers**: for example, $5 - 2x + 3x^2$ becomes $3x^2 - 2x + 5$. The coefficient of x is -2 , including its sign.

Nonexamples: $1/x = x^{-1}$ and $\sqrt{x} = x^{1/2}$ are not polynomials in x . Their exponents are not nonnegative integers.

LESSON 23 | ADDITION AND SUBTRACTION

Match like terms before combining

Like terms have exactly the same variable part, including exponents. You may combine $3x^2$ and x^2 , but not $3x^2$ and x .

Add | $(3x^2 - 2x + 5) + (x^2 + 7x - 4)$

$$\begin{aligned} &(3x^2 + x^2) \\ &+ (-2x + 7x) + (5 - 4) \\ &4x^2 + 5x + 1 \end{aligned}$$

Group the quadratic terms, linear terms and constants.

Add coefficients; keep each variable part unchanged.

Subtract | $(5x^2 + x - 3) - (2x^2 - 4x + 7)$

$$\begin{aligned} &5x^2 + x - 3 \\ &- 2x^2 + 4x - 7 \\ &(5 - 2)x^2 \\ &+ (1 + 4)x + (-3 - 7) \\ &3x^2 + 5x - 10 \end{aligned}$$

Distribute -1 to every term in the second parentheses.

Group like terms after changing the signs.

Combine coefficients and constants.

Keep the exponent

$$3x^2 + x^2 = 4x^2, \text{ not } 4x^4.$$

Adding like terms changes their coefficient, not their exponent.

LESSON 23 | KEEP THE STRUCTURE VISIBLE

Missing terms and equivalent forms

Use a zero coefficient to align a missing term. It adds nothing, but helps you keep the columns straight.

Operation	x^2 terms	x terms	Constants
Start	$4x^2$	$0x$	6
Subtract	x^2	$-2x$	1
Result	$3x^2$	$2x$	5

So $(4x^2 + 6) - (x^2 - 2x + 1) = 3x^2 + 2x + 5$. In the middle column, $0x - (-2x) = 2x$.

Distribution works in both directions

$$3x(2x + 3) = 6x^2 + 9x$$

Expand by multiplying $3x$ by both terms.

$$6x^2 + 9x = 3x(2x + 3)$$

Read backward to write a common-factor form.

These are equivalent expressions. The next chapter develops factoring methods in more detail.

A perimeter expression

A rectangle has length $2x + 3$ and width $x + 1$, measured in centimeters, with $x > 0$.

$$P = 2(2x + 3) + 2(x + 1) = 4x + 6 + 2x + 2 = 6x + 8 \text{ cm.}$$

At $x = 2$, its sides are 7 cm and 3 cm, and its perimeter is 20 cm.

LESSON 23 | SHORT CHECKS

Your turn

Write answers in descending powers. Show the changed signs before combining a subtraction.

1. Add $(2x^2 + 3x - 4) + (x^2 - 5x + 9)$.

2. Subtract $(4x^2 - x + 2) - (x^2 + 2x - 6)$.

3. Simplify $(3x^2 + 5) - (x^2 - 2x + 1)$. Include the missing x term in your work.

4. Rewrite $8x^2 + 4x$ as $4x$ multiplied by a bracketed expression. Expand to check.

5. A rectangle has sides $3x + 2$ and $x + 4$ centimeters, with $x > 0$. Write and simplify its perimeter.

6. For $7x^2 - 9$, name the degree, leading coefficient and number of terms.

LESSON 23 | WORKED ANSWERS

Check the coefficients and signs

1. $(2 + 1)x^2 + (3 - 5)x + (-4 + 9) = 3x^2 - 2x + 5$.

2. $4x^2 - x + 2 - x^2 - 2x + 6 = 3x^2 - 3x + 8$. The -6 inside the subtracted polynomial becomes $+6$.

3. $3x^2 + 0x + 5 - x^2 + 2x - 1 = 2x^2 + 2x + 4$.

4. $4x(2x + 1)$. Expanding gives $4x \times 2x + 4x \times 1 = 8x^2 + 4x$.

5. $P = 2(3x + 2) + 2(x + 4) = 6x + 4 + 2x + 8 = 8x + 12$ cm. Keep the contextual restriction $x > 0$.

6. **Degree 2; leading coefficient 7; two terms**, so it is a quadratic binomial. The constant term is -9 .

Explain the most common subtraction error

Subtracting a polynomial means subtracting **all** its terms. Write a second line with no parentheses and the signs changed before you combine anything.

I can...

- ☐ Identify degree and group like terms.
- ☐ Distribute a minus sign across a whole polynomial.
- ☐ Connect expanded and common-factor forms.

Next class: multiply polynomials systematically. Curriculum focus: A.10A, A.10D.