

CHAPTER 02

Algebra I

Linear functions, graphs and data

Classes 7-14 . November 3-December 3, 2026
Tuesdays and Thursdays . 6:00-7:00 PM Central

CONNECT THE RULE TO ITS STORY

$$y = 2x + 3$$

A rule for every allowed input.

$$m = 2$$

The output rises by 2 per input unit.

$$(0, 3)$$

The line crosses the y-axis at 3.

Clear explanations . Worked examples . Accurate graphs
Short practice checks . Worked answers . Chapter revision

[CONTENTS | CLICK A LESSON TO OPEN IT](#)

Your route through Chapter 2

Open your next lesson, or revisit a topic. Each lesson includes explanations, worked examples and short checks with answers.

07	Slope, rate of change and proportional relationships November 3, 2026	3
08	Graphing lines and the week-four progress check November 5, 2026	8
09	Writing linear equations in three forms November 10, 2026	13
10	Parallel, perpendicular, horizontal and vertical lines November 12, 2026	19
11	Linear domains, ranges and direct variation November 17, 2026	24
12	Transformations of the linear parent function November 19, 2026	29
13	Scatterplots, correlation and fitting a line December 1, 2026	35
14	Unit 2 checkpoint and linear-model review December 3, 2026	41

Chapter quick reference: page 45 . Study guidance and sources: page 46

LESSON 7 | MEET THE IDEA

Slope, rate of change and proportional relationships

Tuesday, November 3, 2026 . 6:00–7:00 PM Central

By the end, I can...

Find slope from points, a table, a graph or an equation, and explain what its sign and units mean.

Slope compares the vertical change with the horizontal change. For a straight, nonvertical line, that rate stays constant.

The slope rule

$$m = (y_2 - y_1) / (x_2 - x_1)$$

Think: **change in output / change in input**. Use the same point order on the top and bottom. The denominator cannot be zero.

Example 1 | Use the points (1, 3) and (4, 9)

$$m = (9 - 3) / (4 - 1)$$

Subtract second-point values minus first-point values in both places.

$$m = 6 / 3 = 2$$

The output increases by 2 for each increase of 1 in the input.

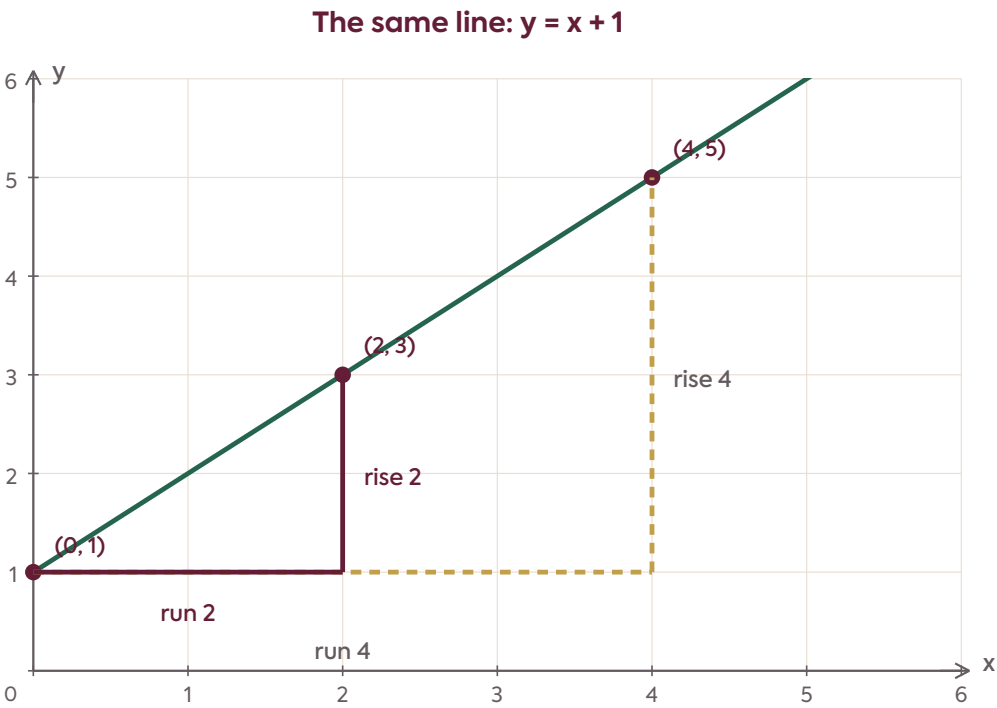
x	1	4	6
y	3	9	13

The table also has slope 2: from $x = 4$ to $x = 6$, $(13 - 9)/(6 - 4) = 4/2 = 2$. Unequal input gaps are fine; compare **ratios**, not just output differences.

LESSON 7 | A VISUAL EXPLANATION

Why the slope stays the same

Draw two right triangles along one straight line. Both have a right angle and the same angle where the line meets a horizontal side, so the triangles are **similar**.



Triangle	Rise / run	Slope
Small, solid sides	2 / 2	1
Large, dashed sides	4 / 4	1

The larger triangle doubles **both** matching side lengths. Its ratio stays 1. Similar triangles explain why choosing different pairs of points on a nonvertical line gives the same slope.

Keep the direction consistent

Going backward changes both signs: $(-2)/(-2) = 1$. Changing the order in only the numerator would give the wrong sign.

LESSON 7 | CONTEXT AND EQUATION FORMS

Give the rate a meaning

Example 2 | A tank holds 18 L at 2 min and 10 L at 6 min

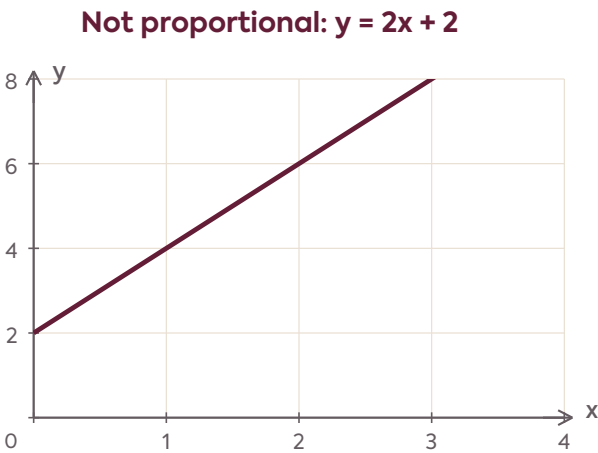
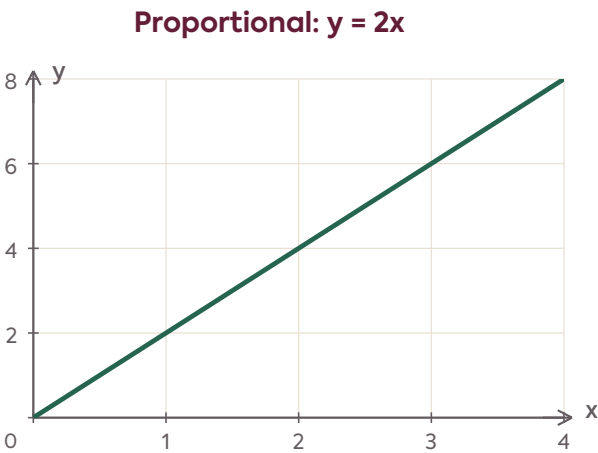
$$m = (10 - 18) / (6 - 2)$$
$$m = -8 / 4 = -2$$

Compare liters with minutes, in the same point order.

The tank loses 2 liters per minute during this constant-rate period.

Equation	Find the slope
$y = 3x + 4$	$m = 3$: the coefficient of x after y is isolated.
$2x + y = 6$	Rewrite as $y = -2x + 6$, so $m = -2$.
$y - 5 = (1/2)(x - 2)$	$m = 1/2$. This point-slope form is developed in Class 9.

A **proportional relationship** has the form $y = kx$ and passes through $(0, 0)$. Its slope k is the unit rate. A fixed starting fee would change the model to $y = mx + b$ with $b \neq 0$.



Horizontal line: slope 0. Vertical line: undefined slope, because the horizontal change is 0. Class 10 explores both cases.

LESSON 7 | SHORT CHECKS

Your turn

Try these before reading the worked answers. Your main homework is in digital practice.

1. Find the slope through (2, 5) and (6, 13).

2. A table pairs $x = 0, 2, 5$ with $y = 7, 3, -3$. Show that both successive intervals have the same slope.

3. Two right triangles along a line have rise/run $3/2$ and $9/6$. Explain why the slopes agree.

4. A tank contains 40 liters at 0 minutes and 28 liters at 3 minutes. Find and interpret the constant rate.

5. For $y = 3x$, list three points including the origin. Is it proportional? What is its slope?

Before you move on

Have you kept the point order consistent? Have you divided by the change in x ? If the quantities have units, have you included them?

LESSON 7 | WORKED ANSWERS AND RECAP

Check your thinking

1. $m = (13 - 5)/(6 - 2) = 8/4 = 2$.
2. First interval: $(3 - 7)/(2 - 0) = -4/2 = -2$. Second: $(-3 - 3)/(5 - 2) = -6/3 = -2$. Both rates match.
3. $9/6 = 3/2$. The larger triangle multiplies both matching sides by 3, so the ratio is unchanged. Both slopes are **1.5**.
4. $(28 - 40)/(3 - 0) = -12/3 = -4$ **liters per minute**. The tank loses 4 liters each minute over this period.
5. Possible points: **(0, 0), (1, 3), (2, 6)**. Yes: it has the form $y = kx$, with **$k = m = 3$** . Its graph is a line through the origin.

Slope describes a change

Positive: y increases as x increases. Negative: y decreases. Zero: y stays constant. Undefined: the line is vertical, so there is no nonzero horizontal change.

I can...

- ☐ Calculate slope from different representations.
- ☐ Explain constant slope using similar triangles.
- ☐ Interpret a rate and recognize a proportional model.

Next class: graph a line and interpret its intercepts. Curriculum focus: A.3A, A.3B, A.2D.

LESSON 8 | MEET THE IDEA

Graphing lines and the week-four progress check

Thursday, November 5, 2026 . 6:00–7:00 PM Central

By the end, I can...

Graph $y = mx + b$ and connect slope, intercepts and zeros to the graph and its context.

The **y-intercept** is where $x = 0$. In $y = mx + b$ it is the point $(0, b)$. Starting there, use the slope to find another point.

Example 1 | Graph $y = 2x - 4$

$b = -4$: plot $(0, -4)$

$m = 2 = 2/1$

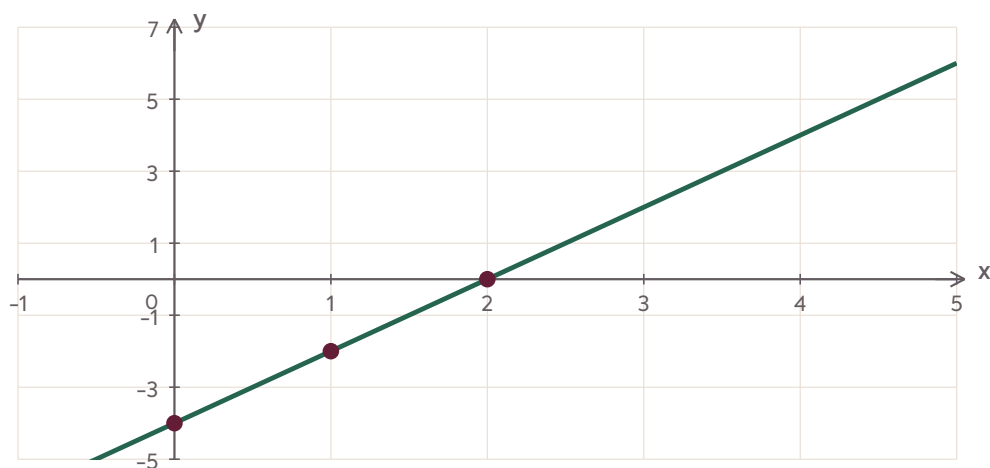
Draw the straight line

The constant gives the starting point on the y-axis.

Move right 1 and up 2 to $(1, -2)$, then to $(2, 0)$.

Check one more point by substituting into the equation.

$y = 2x - 4$



LESSON 8 | READ THE GRAPH

Intercepts tell different stories

Feature	What you set to zero	For $y = 2x - 4$
y-intercept	Set $x = 0$.	$y = -4$; point $(0, -4)$.
x-intercept	Set $y = 0$.	$0 = 2x - 4$ gives $x = 2$; point $(2, 0)$.
Zero of the function	Find the input with output 0.	The zero is the number 2.

Point or number?

The **x-intercept** is **(2, 0)**. The **zero** is **2**. They describe the same crossing, but one is a point and the other is an input value.

Example 2 | Use intercepts to graph $2x + 3y = 6$

$x = 0: 3y = 6, \text{ so } y = 2$	Plot the y-intercept $(0, 2)$.
$y = 0: 2x = 6, \text{ so } x = 3$	Plot the x-intercept $(3, 0)$.
$y = -(2/3)x + 2$	This equivalent rule confirms slope $-2/3$.

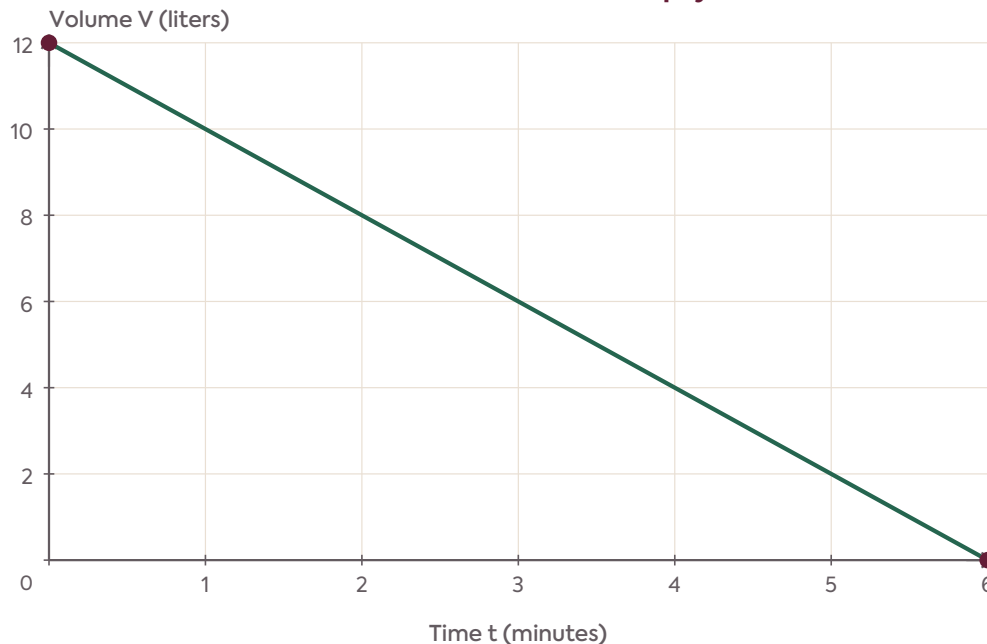
Draw a line through the two intercepts. If the intercepts are the **same** point, as for $y = 2x$, choose another input instead: two copies of the origin are not two different points.

LESSON 8 | A GRAPH WITH UNITS

Read a line in a real situation

A tank starts with 12 liters and drains at 2 liters per minute. Let t be minutes and V be liters remaining. The model is $V = 12 - 2t$, for $0 \leq t \leq 6$.

The tank drains until it is empty



(0, 12): the starting volume is 12 liters.

Slope -2: volume drops by 2 liters each minute.

(6, 0): the tank is empty after 6 minutes. Continuing the line would predict negative water, so the situation stops here.

Today also includes the week-four progress check

Your teacher supplies a separate foundation assessment with fresh questions covering the same baseline skills. Today's new graphing work is reported separately. This PDF contains study notes and practice, not that assessment.

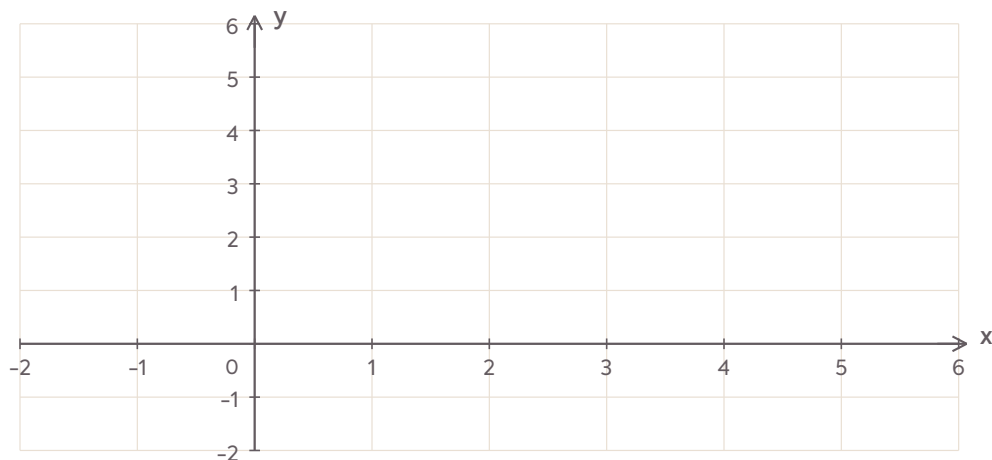
LESSON 8 | SHORT GRAPHING CHECKS

Your turn

Use the grid for Question 1. Try the questions before opening the answers.

- Graph $y = -x + 3$. Mark both intercepts and name the zero.

Your graph



- For $y = (1/2)x + 2$, give the y-intercept and two other points. What is the x-intercept?

- A battery model is $B = 90 - 15t$, for $0 \leq t \leq 6$. B is charge percentage and t is hours. Interpret both intercepts.

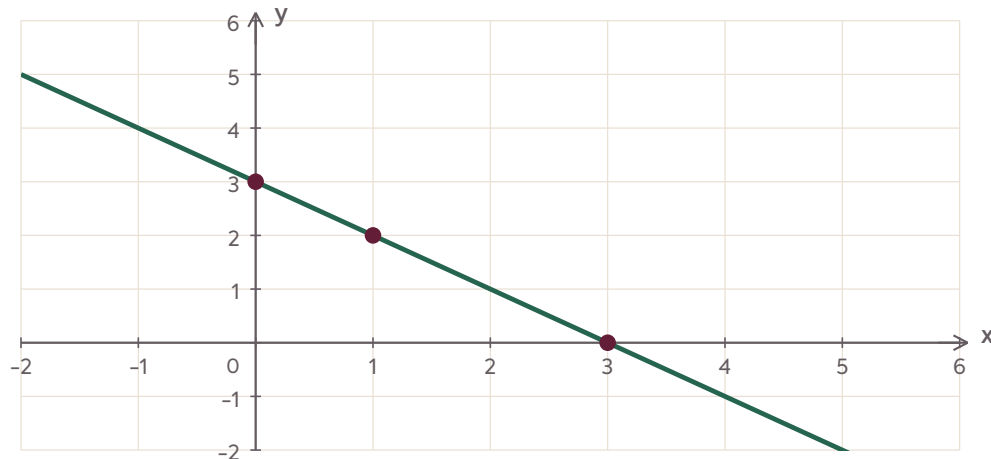
- A student starts the graph of $y = 2x + 3$ at $(3, 0)$. Explain the mistake and give the correct starting point.

LESSON 8 | WORKED ANSWERS

Check your graph and its meaning

1. The intercepts are **(0, 3)** and **(3, 0)**. The zero is **3**. A third check point is **(1, 2)**.

Answer 1: $y = -x + 3$



2. The y-intercept is **(0, 2)**. Possible points: **(2, 3)** and **(4, 4)**. Set $y = 0$: $0 = (1/2)x + 2$, so $x = -4$. The x-intercept is **(-4, 0)**.

3. **(0, 90)** means 90% charge at the start. **(6, 0)** means the model reaches 0% after 6 hours. It is not intended to predict negative charge.

4. The constant 3 gives the **y**-intercept: **(0, 3)**. In **(3, 0)**, the input rather than the output is 3.

I can...

- ☐ Graph a line from its rule.
- ☐ Distinguish an intercept point from a zero.

Next class: write a line equation from given information. Curriculum focus: A.3C.

LESSON 9 | MEET THE IDEA

Writing linear equations in three forms

Tuesday, November 10, 2026 . 6:00–7:00 PM Central

By the end, I can...

Write an equation from a point and slope, two points, a table, a graph or words, then change its form without changing its line.

Form	Equation	Useful when...
Slope-intercept	$y = mx + b$	You know slope m and y -intercept b .
Point-slope	$y - y_1 = m(x - x_1)$	You know m and a point (x_1, y_1) .
Standard	$Ax + By = C$	You want to organize both variables on the left.

In standard form, A and B cannot both be zero. We will usually use integer coefficients. A , B and C here are constants; they are not point coordinates.

Example 1 | Slope 3 through (2, 7)

$y - 7 = 3(x - 2)$

Place the point coordinates in point-slope form.

$y - 7 = 3x - 6$

Distribute the slope across the parentheses.

$y = 3x + 1$

Add 7. The y -intercept is $(0, 1)$.

Check the given point: $3(2) + 1 = 7$. Standard form is $3x - y = -1$.

LESSON 9 | POINTS AND TABLES

Two points give you a route

Example 2 | Through (-1, 5) and (3, -3)

$$m = (-3 - 5)/(3 - (-1))$$

$$m = -8/4 = -2$$

$$y - 5 = -2(x + 1)$$

$$y = -2x + 3$$

Keep the order consistent in both differences.

Use either point with this slope.

$x - (-1)$ becomes $x + 1$.

Distribute, then add 5.

Standard form: $2x + y = 3$. Both points satisfy this equation.

A table follows the same method

x	0	2	5
y	4	10	19

The slopes are $(10 - 4)/(2 - 0) = 3$ and $(19 - 10)/(5 - 2) = 3$. Since $x = 0$ gives $y = 4$, the equation is **$y = 3x + 4$** .

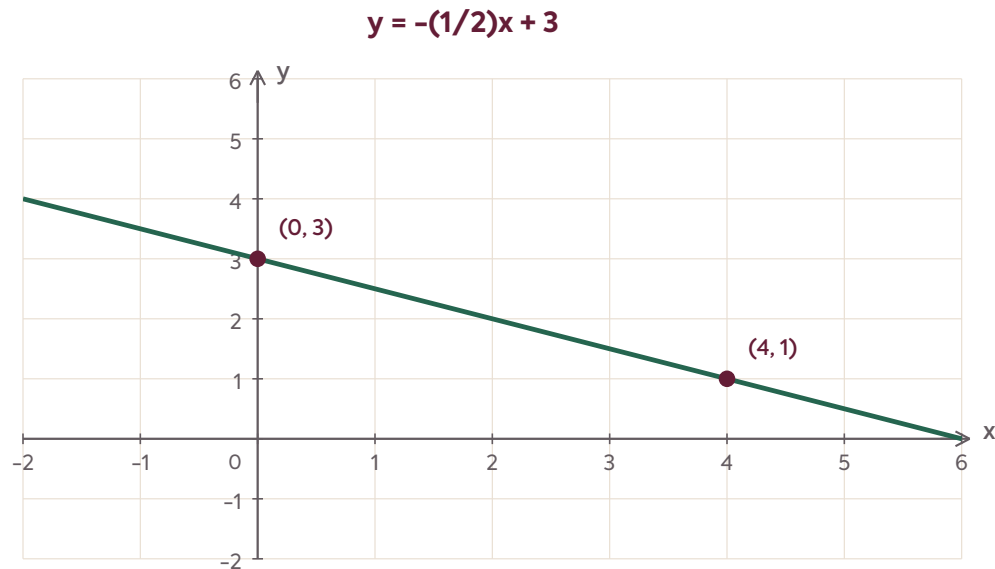
Do not guess b from any output

b is the output at **$x = 0$** . If your table has no zero input, use a known point in $y = mx + b$ and solve for b.

LESSON 9 | REPRESENT THE SAME RELATIONSHIP

From a graph or a sentence

Use exact labeled points on a supplied graph. Here $(0, 3)$ and $(4, 1)$ give $m = (1 - 3)/(4 - 0) = -1/2$ and $b = 3$.



Example 3 | Bike rental: \$5 to start plus \$2 per hour

$$C = 2h + 5$$

h is rental time in hours; C is the total cost in dollars.

$$C = 2(3) + 5 = 11$$

For 3 hours, the model gives a total cost of \$11.

The slope is \$2 per hour. The fixed starting charge is \$5. Assume time is billed proportionally, with $h \geq 0$.

A sensible equation answers two questions: **what changes per unit?** and **what is the starting amount?** Define the variables and their units before substituting.

LESSON 9 | EQUIVALENT EQUATIONS

Change the form, keep the line

Example 4 | Find the slope in $6x + 3y = 12$

$3y = -6x + 12$
 $y = -2x + 4$
 $m = -2; b = 4$

Subtract 6x from both sides.

Divide every term by 3.

Read the slope only after y is isolated.

The same line	What this form reveals
$y = -2x + 4$	Slope -2 and y-intercept (0, 4).
$y - 2 = -2(x - 1)$	Slope -2 and a point (1, 2).
$2x + y = 4$	Both variables grouped on the left.

Two checks serve different purposes

Substituting a given point checks that the line goes through it. To show two equations describe the **same entire line**, rearrange one into the other with valid algebra. Sharing just one point is not enough.

Most of today’s examples are nonvertical lines. A vertical line has the form **x = constant**, which cannot be written as $y = mx + b$. Class 10 handles that special case.

LESSON 9 | SHORT CHECKS

Your turn

1. Write the line with slope 2 through (3, 8). Give point-slope, slope-intercept and standard forms.

2. Find the equation through (-2, 7) and (2, -1).

3. A table gives (1, 5), (3, 11), (4, 14). Find its linear rule.

4. Convert $3x + 2y = 8$ to slope-intercept form. State the slope and y-intercept.

5. An art studio charges a \$7 entry fee plus \$4 per project. Write a cost rule, define the variables and find the cost of 3 projects.

6. Do $y - 5 = 2(x - 1)$ and $y = 2x + 3$ represent the same line? Show why.

Choose a starting point

Point and slope: use point-slope form. Two points: find m first. Table or graph: find m and a point.
Words: identify the rate and the starting value.

LESSON 9 | WORKED ANSWERS AND RECAP

Check your thinking

1. $y - 8 = 2(x - 3)$; expanding gives $y = 2x + 2$. One standard form is $2x - y = -2$. Check: $2(3) + 2 = 8$.
2. $m = (-1 - 7)/(2 - (-2)) = -8/4 = -2$. Using $(2, -1)$: $-1 = -2(2) + b$, so $b = 3$. The equation is $y = -2x + 3$.
3. $m = (11 - 5)/(3 - 1) = 3$. Then $5 = 3(1) + b$ gives $b = 2$. The rule is $y = 3x + 2$; it also gives 14 when $x = 4$.
4. $2y = -3x + 8$, so $y = -(3/2)x + 4$. Slope: $-3/2$. y-intercept: $(0, 4)$.
5. Let n be the number of projects and C the cost in dollars. $C = 4n + 7$, where n is a nonnegative whole number. For $n = 3$: $C = 12 + 7 = \$19$.
6. Yes. Distribute: $y - 5 = 2x - 2$. Add 5: $y = 2x + 3$. Valid rearrangement shows that all solutions agree.

I can...

- ☐ Choose a form that matches the information.
- ☐ Write a rule from points, a graph, a table or words.
- ☐ Show that two line equations are equivalent.

Next class: compare parallel and perpendicular lines. Curriculum focus: A.2B, A.2C, A.3A.

LESSON 10 | MEET THE IDEA

Parallel, perpendicular, horizontal and vertical lines

Thursday, November 12, 2026 . 6:00–7:00 PM Central

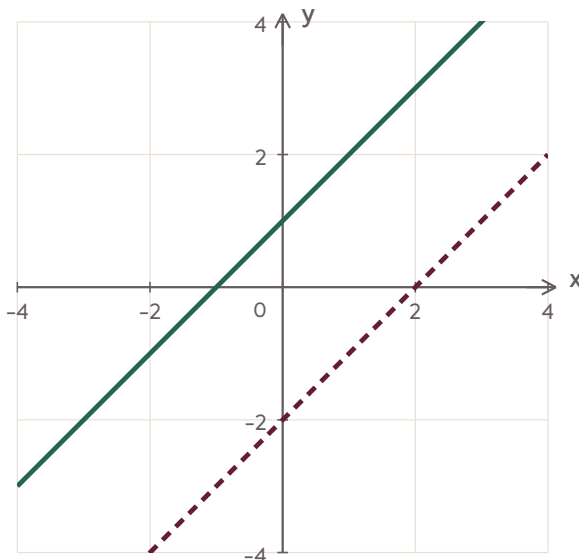
By the end, I can...

Write parallel and perpendicular lines through a point, including horizontal and vertical cases.

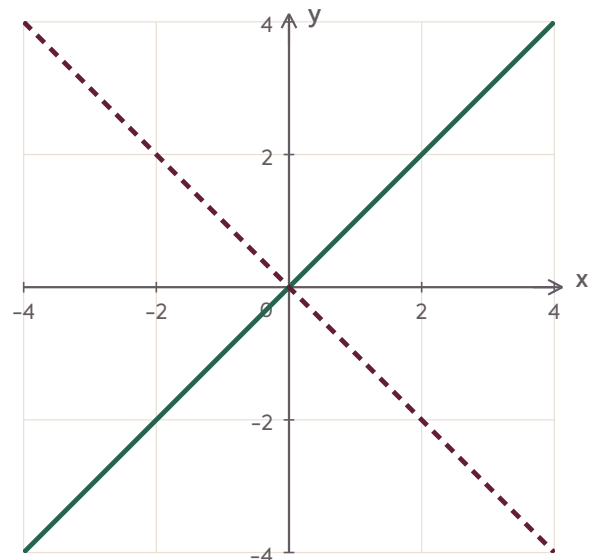
Parallel lines are distinct lines that never meet. Nonvertical parallel lines have the same slope.

Perpendicular lines meet at a right angle (90°).

Parallel lines



Perpendicular lines



Left: solid $y = x + 1$ and dashed $y = x - 2$ both have slope 1.

Right: solid $y = x$ and dashed $y = -x$ have slopes 1 and -1 .

The nonzero-slope rule

For perpendicular lines with finite, nonzero slopes: $m_2 = -1/m_1$. Their slopes multiply to -1 . Horizontal and vertical lines form their own special perpendicular pair.

LESSON 10 | WORKED EXAMPLES

Keep or change the slope first

Example 1 | Parallel to $y = 2x + 1$ through $(3, 4)$

$$m = 2$$

Parallel means keep the slope.

$$y - 4 = 2(x - 3)$$

Use the new point, not the old y-intercept.

$$y = 2x - 2$$

Distribute and simplify.

Check: $2(3) - 2 = 4$. The new intercept differs, so these are distinct parallel lines.

Example 2 | Perpendicular to $y = 2x + 1$ through $(2, 3)$

$$m = -1/2$$

Take the negative reciprocal of 2.

$$y - 3 = -(1/2)(x - 2)$$

Use the perpendicular slope with the new point.

$$y = -(1/2)x + 4$$

Distribute, then add 3.

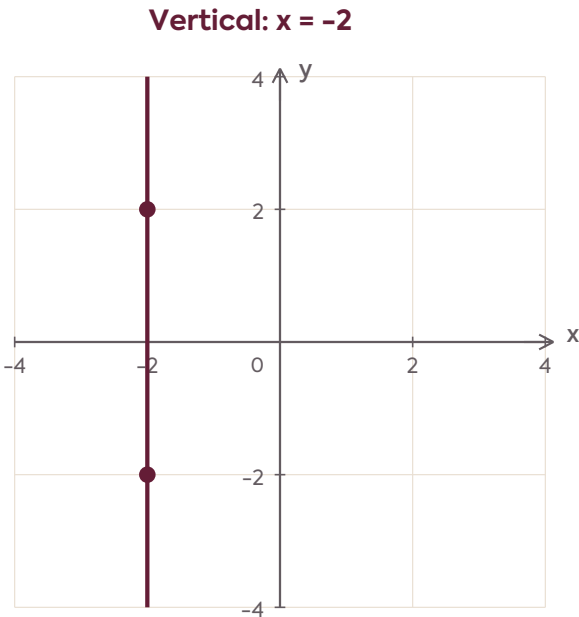
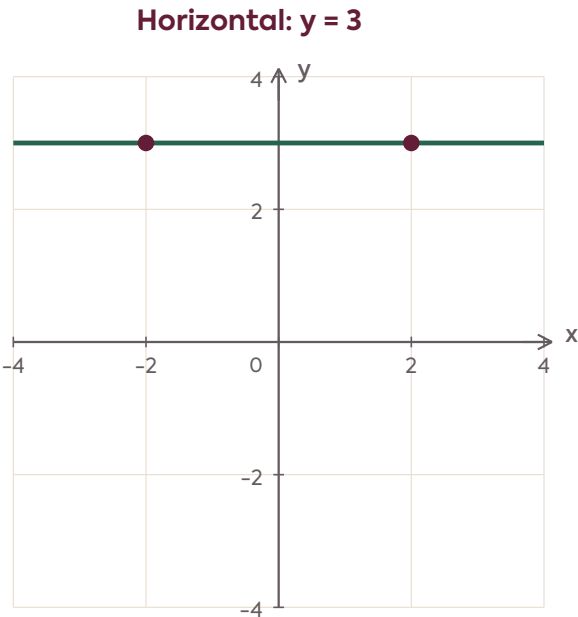
Check the slope product: $2(-1/2) = -1$. Check the point: $-(1/2)(2) + 4 = 3$.

Do not copy the whole equation

A parallel line keeps the **slope**, but its intercept usually changes. If the supplied point is already on the given line, keeping that slope reproduces the same line, not a distinct parallel line.

LESSON 10 | THE SPECIAL CASES

Flat is different from vertical



Line type	What stays fixed	Slope
Horizontal: $y = b$	Every point has the same y-coordinate.	0: rise is 0.
Vertical: $x = a$	Every point has the same x-coordinate.	Undefined: run is 0.

Through $(4, -1)$, the horizontal line is $y = -1$; the vertical line is $x = 4$. They are perpendicular to each other. The first is parallel to the x-axis; the second is parallel to the y-axis.

Zero is a number; undefined is not

For a horizontal line, $0/4 = 0$. For a vertical line, $4/0$ is undefined. A vertical line also fails the function test for y as a function of x .

LESSON 10 | SHORT CHECKS

Your turn

1. Write a line parallel to $y = 3x - 2$ through $(1, 5)$.

2. Write a line perpendicular to $y = 2x + 7$ through $(4, 1)$.

3. Write the horizontal and vertical lines through $(-3, 2)$. State each slope.

4. Write the line through $(5, -2)$ that is perpendicular to $y = 4$.

5. Are $2x + y = 6$ and $y = (1/2)x - 1$ perpendicular? Explain using slopes.

6. A student says the slope through $(2, 1)$ and $(2, 5)$ is 0. Explain the error.

A reliable order

Identify the given slope. Decide the new slope or special line type. Use the point. Finally, check that point in your equation.

LESSON 10 | WORKED ANSWERS AND RECAP

Check your thinking

1. Keep $m = 3$. Then $y - 5 = 3(x - 1)$, giving $y = 3x + 2$.
2. Use $m = -1/2$. Then $y - 1 = -(1/2)(x - 4)$, giving $y = -(1/2)x + 3$. At $x = 4$, $y = 1$.
3. Horizontal: $y = 2$, slope **0**. Vertical: $x = -3$, slope **undefined**.
4. $y = 4$ is horizontal, so its perpendicular line is vertical. Through the given point, that line is $x = 5$.
5. Yes. Rewrite the first as $y = -2x + 6$. The slopes are -2 and $1/2$, and $(-2)(1/2) = -1$.
6. $m = (5 - 1)/(2 - 2) = 4/0$, which is **undefined**. Both points have $x = 2$, so the line is vertical.

Remember the pair

Horizontal lines have equations $y = a$ constant. Vertical lines have equations $x = a$ constant. They meet at 90° , even though the negative-reciprocal formula cannot be applied to slope 0.

I can...

- ☐ Write parallel and perpendicular equations.
- ☐ Handle axis-parallel lines correctly.
- ☐ Check the new line with the given point.

Next class: decide which inputs and outputs make sense. Curriculum focus: A.2E, A.2F, A.2G.

LESSON 11 | MEET THE IDEA

Linear domains, ranges and direct variation

Tuesday, November 17, 2026 . 6:00–7:00 PM Central

By the end, I can...

Choose sensible inputs and outputs, distinguish continuous values from counts, and build direct variation rules.

Word	Meaning	Question to ask
Domain	The allowed inputs.	Which x-values can I use?
Range	The outputs produced by those inputs.	Which y-values actually occur?
Continuous	All values across an interval, including fractions.	Is this a measurement?
Discrete	Separate allowed values.	Is this a whole-number count?

Example 1 | $y = 2x + 1$ with no stated restriction

Domain: all real numbers

Any real input can be multiplied by 2 and increased by 1.

Range: all real numbers

Every real output is reached: solve $x = (y - 1)/2$.

Remember the horizontal exception

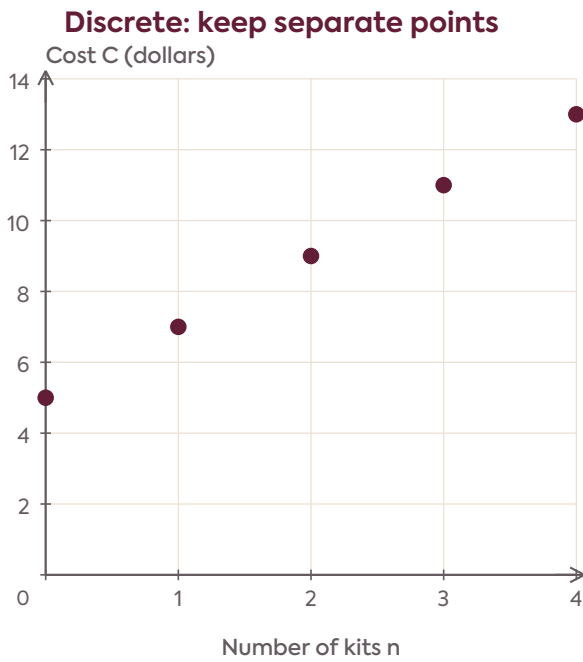
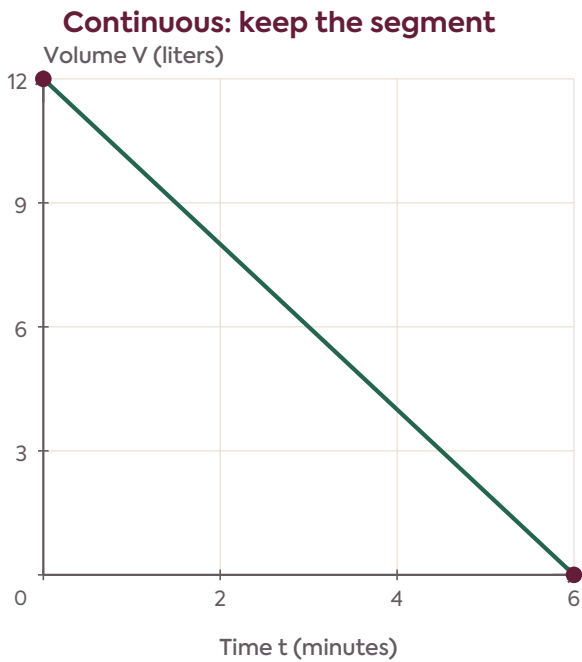
For $y = 5$ with no input restriction, the domain is all real numbers, but the range is **only 5**. A constant line does not reach every output.

LESSON 11 | CONTINUOUS AND DISCRETE MODELS

A situation can restrict a line

Tank: $V = 12 - 2t$ from the start until empty. Time is continuous: $0 \leq t \leq 6$. The volume range is $0 \leq V \leq 12$.

Club: $C = 5 + 2n$ for buying up to four project kits. n must be a whole number: $n = 0, 1, 2, 3, 4$. The costs are **5, 7, 9, 11, 13 dollars**.



Bounds alone are not enough

For the club, $0 \leq n \leq 4$ must be accompanied by “ **n is a whole number**”. Similarly, $5 \leq C \leq 13$ alone would wrongly suggest that \$6 is possible. List the outputs or describe their \$2 steps.

The tank graph includes fractional minutes such as 2.5. The kit graph excludes fractional counts such as 2.5. That is why one graph is joined and the other is not.

LESSON 11 | WRITE $y = kx$

Direct variation: one constant rate

In **direct variation**, $y = kx$. For each nonzero input, y/x has the same value k . The unrestricted graph is a straight line through the origin. Real contexts may use only part of that line.

Example 2 | Fabric costs \$14 for 4 meters, with no fixed fee

$$k = 14/4 = 3.5$$

Divide cost by length: \$3.50 per meter.

$$C = 3.5l$$

l is length in meters; C is cost in dollars.

$$C = 3.5(6) = 21$$

Six meters cost \$21 under this constant-price model.

For continuous purchases, $l \geq 0$. The model gives $C = 0$ at $l = 0$.

Example 3 | y varies directly with x ; $y = 18$ when $x = 6$

$$k = 18/6 = 3$$

Find the constant before finding a new output.

$$y = 3x$$

Write the complete relationship.

$$27 = 3x, \text{ so } x = 9$$

When $y = 27$, solve for the input.

A constant slope is not the whole test

$C = 3.5l + 5$ has constant slope, but the \$5 fixed fee means it is **not** direct variation. Its unrestricted graph misses the origin.

LESSON 11 | SHORT CHECKS

Your turn

1. State the domain and range of $y = -3x + 7$ when there are no restrictions.

2. For $y = 2x + 1$, the domain is $x = 0, 1, 2, 3$ only. List the range. Should you join these points on a graph?

3. A tank model is $V = 20 - 4t$, for $0 \leq t \leq 5$. Time t is continuous. Give the domain and range, with units.

4. y varies directly with x . When $x = 5$, $y = 20$. Find the rule and find y when $x = 8$.

5. A club charges \$5 to join plus \$2 per visit. Is the cost directly proportional to the number of visits? Explain.

6. For $y = 4$ with any real input, what is the range?

Say both parts

State the bounds **and** the kind of values allowed. Use a list when an inequality would include outputs that the context cannot produce.

LESSON 11 | WORKED ANSWERS AND RECAP

Check your thinking

- 1. Domain: all real numbers. Range: all real numbers.** The nonzero slope means every real output can be reached by some real input.
- 2.** The range is **1, 3, 5, 7**. Do **not** join the points: no inputs between the listed values are allowed.
- 3.** Domain: **$0 \leq t \leq 5$ minutes**, including fractional times. Range: **$0 \leq V \leq 20$ liters**. At $t = 0$, $V = 20$; at $t = 5$, $V = 0$.
- 4.** $k = 20/5 = 4$, so **$y = 4x$** . When $x = 8$, $y = 4(8) = 32$.
- 5. No.** The rule is $C = 2n + 5$, so cost is \$5 when the visit count is 0. A direct variation rule would have the form $C = kn$, with no added fixed fee.
- 6.** The range contains **only 4**. Changing the input never changes the output.

I can...

- ☐ State domain and range accurately.
- ☐ Distinguish measurements from discrete counts.
- ☐ Find k and solve a direct variation problem.

Next class: transform the parent line $f(x) = x$. Curriculum focus: A.2A, A.2D.

LESSON 12 | MEET THE IDEA

Transformations of the linear parent function

Thursday, November 19, 2026 . 6:00–7:00 PM Central

By the end, I can...

Predict and explain the effects of $af(x)$, $f(x) + d$, $f(x - c)$ and $f(bx)$ on the parent function $f(x) = x$.

The **parent function** is the starting rule. For this lesson it is **$f(x) = x$** : its output equals its input, its slope is 1, and it passes through the origin.

Change	What is being changed?	Example when $f(x) = x$
$af(x)$	Multiply the output by a .	$2f(x) = 2x$
$f(x) + d$	Add d to the output.	$f(x) + 3 = x + 3$
$f(x - c)$	Replace the input by $x - c$.	$f(x - 3) = x - 3$
$f(bx)$	Multiply the input by b before applying f .	$f(2x) = 2x$

Inside or outside?

An outside operation acts on the **output**. An inside operation changes the **input** sent into f . Write the new rule first; then check a point.

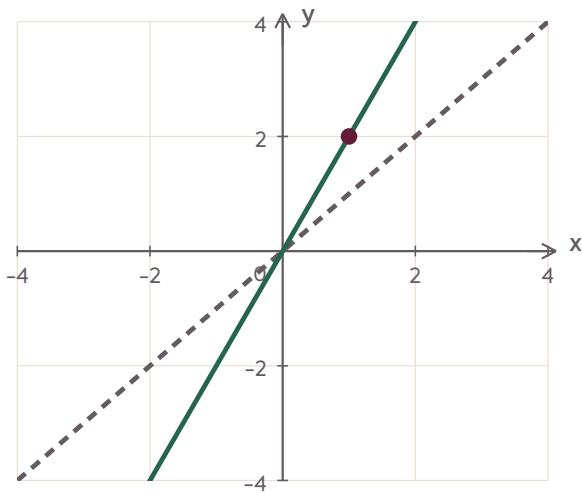
The letters a , b , c and d in this lesson are transformation parameters. The b inside $f(bx)$ is not the y -intercept symbol from $y = mx + b$.

LESSON 12 | VERTICAL CHANGES

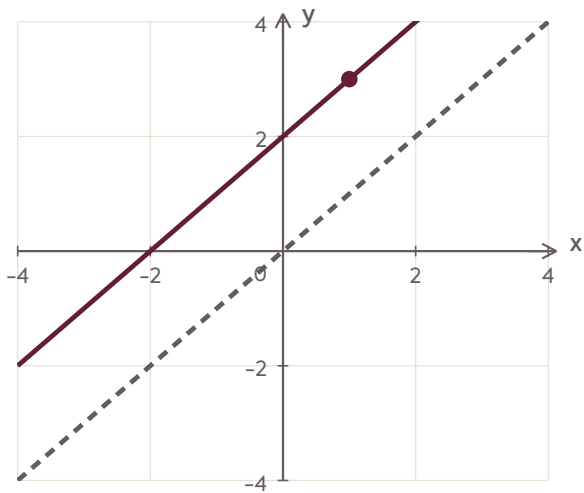
Changes to the output

$af(x)$: multiply every y-coordinate by a . If $|a| > 1$, stretch vertically; if $0 < |a| < 1$, compress vertically. If $a < 0$, also reflect across the x-axis. Here $|a|$ means the size of a without its sign.

Vertical stretch: $2f(x) = 2x$



Shift up: $f(x) + 2 = x + 2$



In both graphs, the dashed line is $f(x) = x$. For $2f(x)$, the parent point $(1, 1)$ becomes **$(1, 2)$** . For $f(x) + 2$, it becomes **$(1, 3)$** .

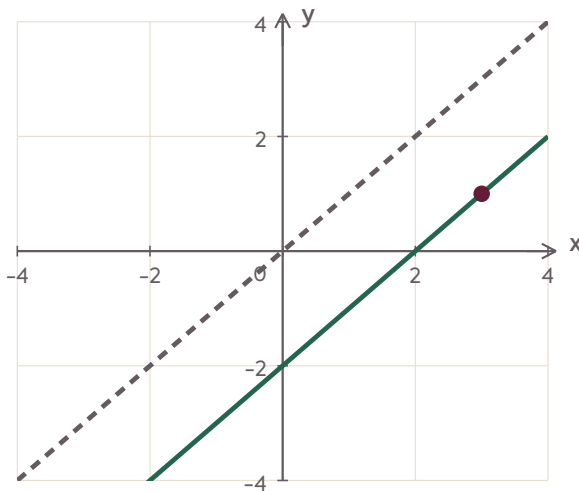
New rule	Effect on the parent graph
$(1/2)f(x) = (1/2)x$	Vertical compression by factor $1/2$.
$-2f(x) = -2x$	Vertical stretch by 2 and reflection across the x-axis.
$f(x) - 3 = x - 3$	Shift down 3 units; slope remains 1.

LESSON 12 | HORIZONTAL CHANGES

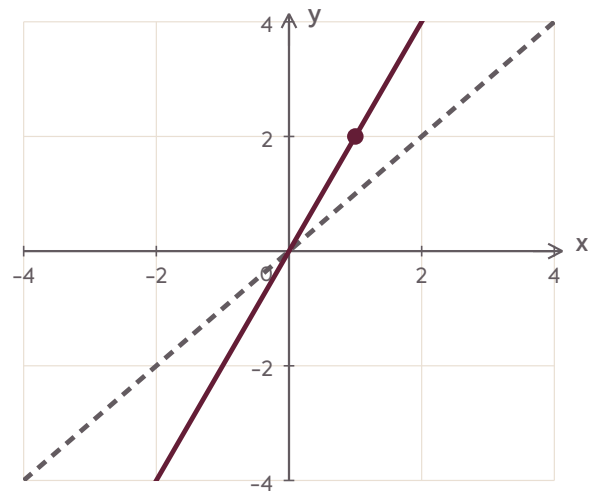
Changes inside the function

$f(x - c)$: shift right c units when $c > 0$; shift left $|c|$ units when $c < 0$. The minus sign inside is why $f(x - 2)$ moves the parent graph **right** by 2.

Shift right: $f(x - 2) = x - 2$



Input scale: $f(2x) = 2x$



For $f(x - 2)$, the parent point (1, 1) moves to **(3, 1)**. For $f(2x)$, the parent point (2, 2) moves to **(1, 2)**: the new input only needs to be half as large.

The factor inside works reciprocally

For $f(bx)$, with $b \neq 0$, each point $(u, f(u))$ moves to **$(u/b, f(u))$** . The horizontal scale factor is $1/|b|$. If b is negative, also reflect across the y-axis.

So $f((1/2)x)$ stretches horizontally by 2, while $f(-2x)$ compresses horizontally by $1/2$ and reflects across the y-axis.

LESSON 12 | WHAT IS SPECIAL ABOUT $f(x) = x$?

Different moves can give the same line

Two descriptions	Both simplify to...	Why they agree here
$2f(x)$ and $f(2x)$	$2x$	The parent simply returns its input.
$f(x) - 3$ and $f(x - 3)$	$x - 3$	A downward shift and a rightward shift produce the same line.
$-f(x)$ and $f(-x)$	$-x$	Reflecting this parent across either axis produces the same line.

Example | Track two different parent points

$2f(x)$: (1, 1) becomes (1, 2)	Multiply the output by 2; the input is unchanged.
$f(2x)$: (2, 2) becomes (1, 2)	Divide the input coordinate by 2; the output is unchanged.

The point moves differ, but the resulting set of points is the same line $y = 2x$.

Do not invent a visual difference

If two rules simplify to the same equation, their graphs coincide. Explain the operation accurately, then say that the resulting line is the same. This coincidence depends on the parent function.

Zero parameters: $0f(x)$ is $y = 0$. Also $f(0x) = f(0) = 0$ for this parent. Do not describe $b = 0$ as scaling by $1/b$; that division is undefined.

LESSON 12 | SHORT CHECKS

Your turn

Use $f(x) = x$ throughout. State the new rule as well as the described transformation.

1. Write $3f(x)$. Where does the parent point $(2, 2)$ move?

2. Write $f(x) - 4$. What happens to the slope and y-intercept?

3. Write $f(x + 2)$. Is the described horizontal shift left or right?

4. Write $f(2x)$. Where does the parent point $(4, 4)$ move? Give the horizontal scale factor.

5. Write $-(1/2)f(x)$. Describe both effects on the output.

6. Do $f(x - 3)$ and $f(x) - 3$ have different graphs? Explain.

One point can reveal the rule

For $g(x) = f(x - 2)$, try $x = 2$: $g(2) = f(0) = 0$. This checks that the old origin has moved to $(2, 0)$.

LESSON 12 | WORKED ANSWERS AND RECAP

Check your thinking

1. $y = 3x$, a vertical stretch by factor 3. The parent point (2, 2) moves to **(2, 6)**.
2. $y = x - 4$, a shift down 4. The slope stays 1; the y-intercept becomes **(0, -4)**.
3. $y = x + 2$. Since $x + 2 = x - (-2)$, the horizontal description is a shift **left 2**.
4. $y = 2x$. The parent point (4, 4) moves to **(2, 4)**. The horizontal scale factor is $\frac{1}{2}$, a compression.
5. $y = -(1/2)x$. Compress vertically by $\frac{1}{2}$ and reflect across the **x-axis**.
6. **No**. Both rules simplify to $y = x - 3$. One describes a rightward shift of this parent; the other describes a downward shift. The final line is identical.

I can...

- ☐ Distinguish input changes from output changes.
- ☐ Use reciprocal horizontal scale factors.
- ☐ Recognize when two transformations give the same line.

Next class: explore scatterplots and fit a line to data. Curriculum focus: A.3E.

The next scheduled class is Tuesday, December 1, after the November break.

LESSON 13 | MEET THE IDEA

Scatterplots, correlation and fitting a line

Tuesday, December 1, 2026 . 6:00–7:00 PM Central

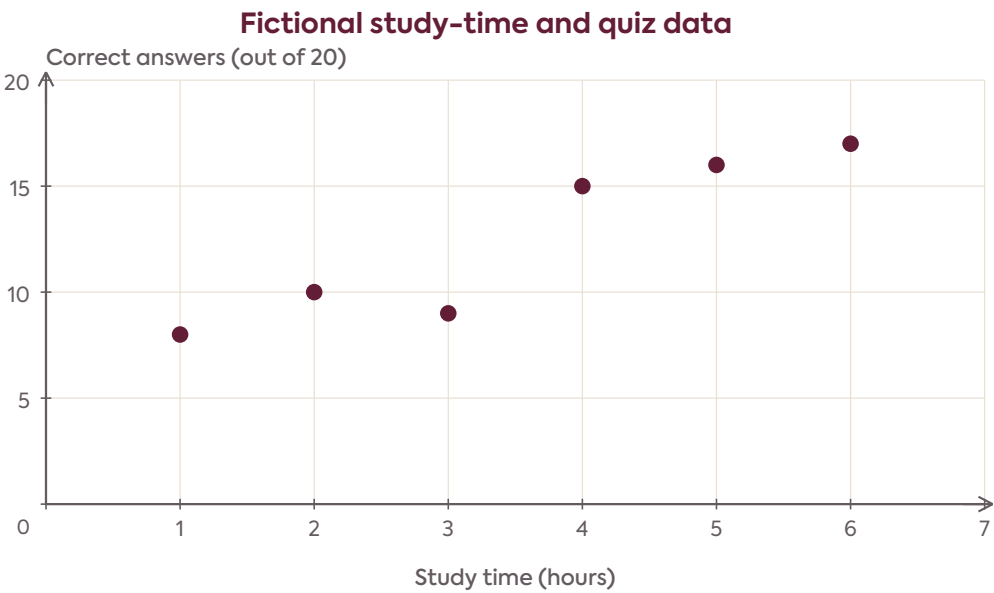
By the end, I can...

Build a scatterplot, fit and use a line, interpret correlation, and explain why association alone does not prove causation.

A **scatterplot** displays paired numerical data. Each dot represents one pair, with the explanatory variable on the x-axis and the response on the y-axis. Look for direction, shape, strength and unusual points.

Student	A	B	C	D	E	F
Study hours x	1	2	3	4	5	6
Correct out of 20, y	8	10	9	15	16	17

These six students and results are **fictional**, created for learning graphing and regression. They are not Cambridge Veritas outcome data.



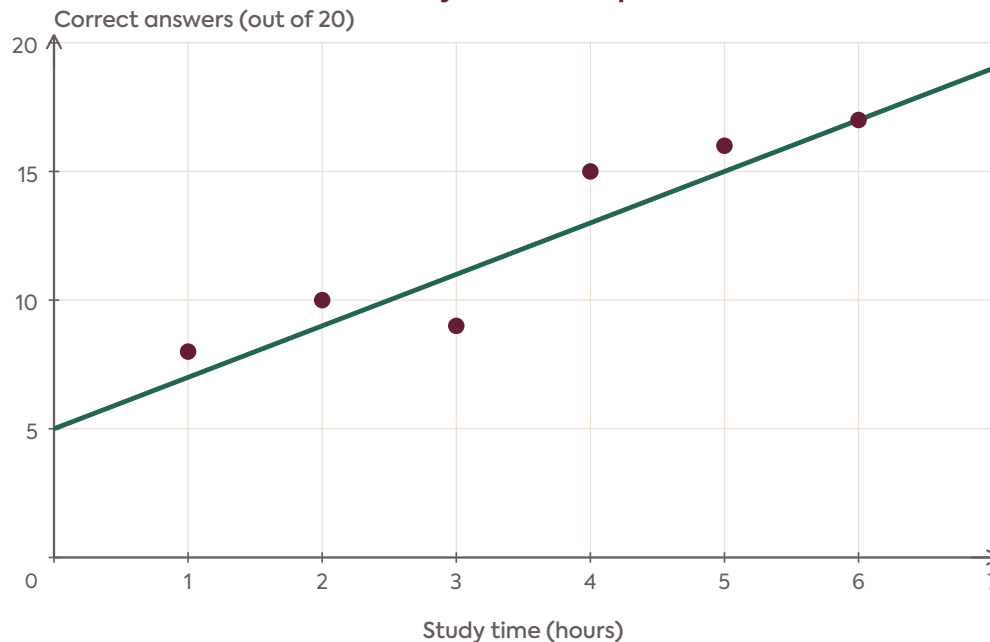
Each dot is a different student. Do not join the dots in time order. The pattern trends upward, but it is not an exact straight-line function.

LESSON 13 | APPROXIMATION AND PREDICTION

Fit a sensible line by eye

Sketch a line that follows the overall trend, with points reasonably balanced above and below it. It does not need to pass through every point. Different sensible hand-fits can be acceptable.

Fictional study-time and quiz data



One possible hand-fit | $y = 2x + 5$

$$m = (15 - 5) / (5 - 0) = 2$$

$$y = 2(4.5) + 5 = 14$$

Use two convenient points on your drawn line: (0, 5) and (5, 15).

At 4.5 hours, this hand-fit predicts about 14 correct answers.

The points used to calculate a hand-fit slope can be points on the fitted line; they do not have to be original data points.

A prediction has limits

4.5 hours is inside the observed 1–6 hour range: **interpolation**. Using 12 hours is **extrapolation**, outside that range. The hand-fit would give 29 correct answers out of 20, which is impossible.

LESSON 13 | REGRESSION AND CORRELATION

Run a fit with technology

Use the graphing or statistics tool prepared by your teacher. Enter the opening data table yourself; the supplied values below are for checking afterward.

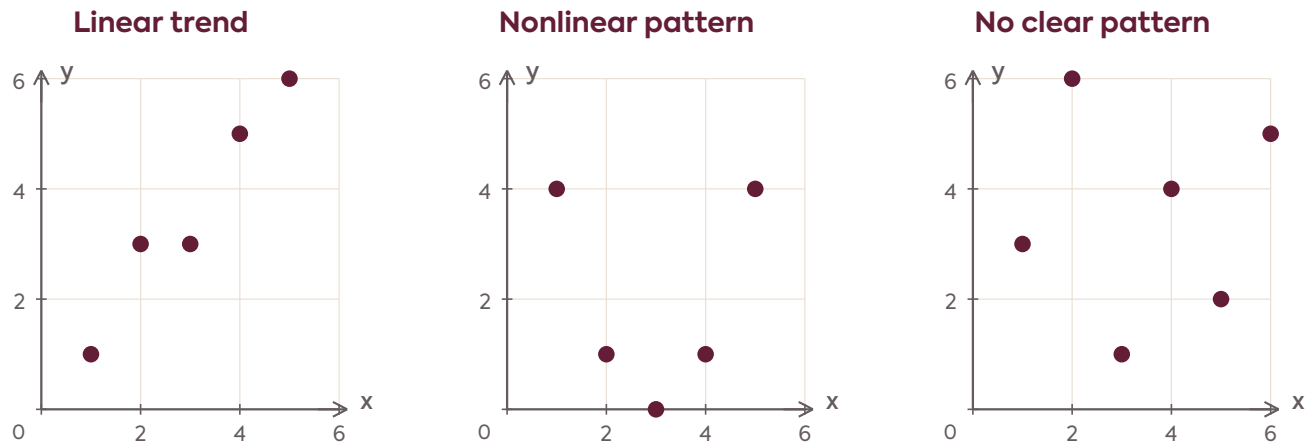
Step	What to do
1. Enter pairs	Put study hours in the x column and correct answers in the y column. Keep each student's values on the same row.
2. Plot	Show the six points and compare them with the opening scatterplot. Fix any entry or axis errors first.
3. Fit a line	Choose linear regression in the form $y = mx + b$. Record both coefficients.
4. Find r	Display the Pearson correlation coefficient r. If your tool shows only R^2 , use its correlation function or your teacher's prepared tool.
5. Interpret	Use the equation to predict and the graph to judge whether a linear fit is sensible.

Expected check valuesRegression: $y \approx 1.97143x + 5.60000$ Pearson correlation: $r \approx 0.93681$ At $x = 4.5$, the predicted value is **about 14.47**, or roughly 14–15 correct answers.

Linear regression chooses the line that minimizes the sum of squared **vertical prediction errors**. Keep full calculator precision while predicting, then round the final answer. Do not confuse r with R^2 .

LESSON 13 | CORRELATION IS NOT CAUSATION

Describe the pattern carefully



r measures the direction and strength of a **linear** association. It lies from -1 to 1 , when defined. Values near 1 suggest a strong positive linear pattern; values near -1 suggest a strong negative one.

A small r can hide a curve

The middle plot has a clear U-shaped pattern, even though its Pearson r is 0 . A value near 0 means little linear association; it does not establish that no relationship exists.

For our study dataset, $r \approx 0.937$ indicates a strong positive linear association: more study time tends to accompany more correct answers. It does **not** mean that studying causes 93.7% improvement, or that a student is guaranteed a score.

What else could explain an association?

Students may differ in prior knowledge, sleep or study methods. These can affect the relationship. Another example: hot weather can increase both ice-cream sales and sunhat sales. Buying ice cream has not thereby been shown to cause sunhat purchases.

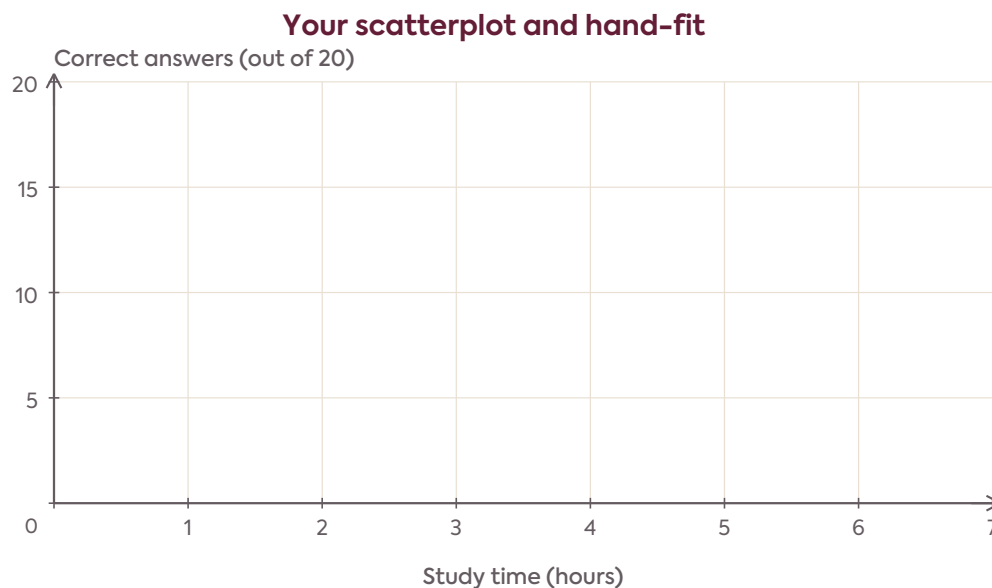
If either variable is constant across all observations, Pearson r is undefined. Always inspect the plot and the context, rather than relying on a single number.

LESSON 13 | SHORT DATA INVESTIGATION

Your turn

Use this lesson's fictional paired data. These checks are practice; use the teacher's tool for the regression task.

1. Plot the six pairs on the grid and sketch a reasonable trend line.



2. Run a linear regression. Record its equation and r to three decimal places.

3. Use the regression to predict the result at 4.5 hours. Would a prediction at 12 hours be equally sensible? Explain.

4. If a different dataset has $r = 0$, must its scatterplot have no pattern? Explain.

5. Does our dataset prove that extra study time causes a higher result? Suggest another factor that might matter.

LESSON 13 | WORKED ANSWERS AND RECAP

Check your investigation

1. Your plot should match the opening scatterplot. One reasonable hand-fit is $y = 2x + 5$, shown in the worked example. A different line is acceptable if it reasonably follows the overall trend; explain your choice.
2. To three decimal places: $y \approx 1.971x + 5.600$ and $r \approx 0.937$. This is a strong positive linear association for these six data pairs.
3. Using the unrounded coefficients: $1.97142857(4.5) + 5.6 \approx 14.47$, about 14–15 correct answers. At 12 hours the fit predicts about **29.26**, exceeding the 20-question maximum. That is an unreasonable extrapolation.
4. **No.** A nonlinear pattern can have $r = 0$; the U-shaped plot in the pattern comparison is an example. r summarizes linear association only.
5. **No.** This is a small fictional observational dataset. Prior knowledge, sleep or study quality could affect results. Association alone does not establish cause and effect.

A useful accuracy check

At $x = 3$, the fitted output is about 11.51, while the observed output is 9. The vertical error, observed minus predicted, is about **-2.51**. That point lies below the regression line.

I can...

- ☐ Construct and describe a scatterplot.
- ☐ Run and interpret a linear regression and r .
- ☐ Question unreasonable predictions and causal claims.

Next class: review the chapter and use your checkpoint evidence. Curriculum focus: A.4A, A.4B, A.4C.

LESSON 14 | REVIEW AND RECONNECT

Unit 2 checkpoint and linear-model review

Thursday, December 3, 2026 . 6:00–7:00 PM Central

Today is for checking and improving

Connect equations, graphs and contexts. These notes are a revision and corrections guide. Your teacher supplies the graded unit checkpoint separately.

Skill	Ask yourself	Revisit
Slope	Did I compare changes in the same order, with units?	Class 7
Graph features	Did I label intercept points and the zero correctly?	Class 8
Line equations	Did I use the given point and rate?	Class 9
Related lines	Did I handle horizontal and vertical cases?	Class 10
Domain and range	Did I state the bounds and the allowed values?	Class 11
Transformations	Did I distinguish input and output changes?	Class 12
Data models	Did I check the plot, prediction and meaning of r ?	Class 13

The one-hour class includes a 20-minute checkpoint. It samples the chapter; your teacher also uses earlier work when deciding what you understand and what to revisit.

LESSON 14 | WORKED REVISION

Connect the representations

Example | A club charges \$6 plus \$3 for each kit

$C = 3n + 6$

Slope 3; intercept (0, 6)

$n = 0, 1, 2, 3, 4$

$C = 6, 9, 12, 15, 18$

n is the kit count; C is cost in dollars.

The rate is \$3 per kit and the fixed fee is \$6.

Suppose no more than four kits may be bought.

These separate outputs form the range; do not join the graph points.

All four transformations: $f(x) = x$

Form and example	Result	Description
$af(x)$: $2f(x)$	$2x$	Multiply the output by 2.
$f(x) + d$: $f(x) + 3$	$x + 3$	Shift up 3.
$f(x - c)$: $f(x - 2)$	$x - 2$	Shift right 2.
$f(bx)$: $f(3x)$	$3x$	Compress horizontally by $1/3$.

Keep the two model questions separate

Does a line fit the pattern?

Inspect the scatterplot and linear correlation.

Has causation been established?

Consider how the data were collected and what other factors might explain the pattern.

LESSON 14 | PREPARE, THEN REFLECT

A short practice check-in

These are revision questions, not the graded checkpoint. Show the reasoning you would want to explain to your teacher.

1. Find the slope and equation through (1, 5) and (3, 9). Give both intercepts.

2. For your rule in Question 1, allow only $x = 0, 1, 2, 3$. List the range.

3. Write the line perpendicular to $y = 2x + 3$ through (0, 1).

4. For $f(x) = x$, simplify $f(x - 4)$ and describe the horizontal transformation. Give another transformation that produces the same line.

5. A fitted line is $y = 2x + 5$ with observed inputs from 1 to 6 and $r = 0.91$. Predict y at $x = 4$ and interpret r . Does r prove causation?

Choose one useful next step

Identify the earliest step that confused you. Revisit its lesson example, cover the next line, predict the step and then check. Finish with a fresh question.

LESSON 14 | WORKED ANSWERS AND REFLECTION

Check, correct and move forward

1. $m = (9 - 5)/(3 - 1) = 2$. Then $5 = 2(1) + b$ gives $b = 3$, so $y = 2x + 3$. Intercepts: **(0, 3)** and **(-1.5, 0)**.
2. The outputs are **3, 5, 7, 9**. The input restriction makes this a discrete set of points.
3. The new slope is $-1/2$, so the line through $(0, 1)$ is $y = -(1/2)x + 1$.
4. **$f(x - 4) = x - 4$** , a rightward shift by 4. Shifting the parent **down 4**, using $f(x) - 4$, produces the same line.
5. At $x = 4$, $y = 2(4) + 5 = 13$. $r = 0.91$ suggests a strong positive **linear association**. It does not prove that changes in x cause changes in y .

My correction plan

Write something specific	Your notes
The step I need to improve	_____
The lesson I will revisit	_____
How I will check a fresh answer	_____

Next chapter begins Tuesday, December 8: linear inequalities in two variables and systems. Keep your slope, equation and graph skills ready.

KEEP THIS PAGE FOR REVISION

Chapter 2 quick reference

Idea	Remember	Example
Slope	Change in y divided by change in x; use the same point order.	$(9 - 5)/(3 - 1) = 2$
Line forms	Use slope-intercept, point-slope or standard form as appropriate.	$y = 2x + 3$; $y - 5 = 2(x - 1)$; $2x - y = -3$
Intercepts	$x = 0$ finds the y-intercept; $y = 0$ finds the x-intercept.	$y = 2x + 3$: (0, 3) and (-1.5, 0)
Related lines	Distinct nonvertical parallel lines share a slope. Perpendicular nonzero slopes multiply to -1.	2 and -1/2 are perpendicular slopes.
Special lines	$y = \text{constant}$ is horizontal; $x = \text{constant}$ is vertical.	$y = 3$: slope 0. $x = 3$: undefined slope.
Domain / range	State possible inputs and outputs, including discrete restrictions.	$x = 0, 1, 2$ in $y = 2x + 1$ gives $y = 1, 3, 5$.
Direct variation	$y = kx$; find $k = y/x$ using a nonzero input.	$y = 12$ at $x = 3$ gives $k = 4$.
Transformations	Outside: output changes. Inside: input changes.	$f(x - 2)$: right 2. $f(2x)$: horizontal factor 1/2.
Data models	Check the scatterplot, fit, prediction range and meaning of r .	Strong correlation alone does not establish causation.

For transformations here, $f(x) = x$. Distinct transformations can yield the same line. For data, distinguish r from R^2 and avoid unsupported extrapolation.

STUDY GUIDANCE AND CURRICULUM REFERENCES

Make these notes work for you

A small routine for each lesson

Before: read the learning goal and recall the previous class.

During: explain the reasons beside the examples and label your graphs.

After: attempt the short checks before reading their answers, then complete the digital practice assigned by your teacher.

Practice and assessment have different jobs

These PDFs include worked answers to their own practice checks. The week-four foundation assessment in Class 8 and graded unit checkpoint in Class 14 are separate teacher-provided materials. Their unpublished questions and keys are not included.

Graphing and data

Draw with labeled axes and sensible scales. For the data lesson, use your teacher's prepared graphing or statistics tool to enter the pairs, run a fit and calculate r . All datasets in this chapter are fictional teaching examples, not reported research or student outcomes.

Curriculum alignment

Standard Algebra I, §111.39(c): A.2A–G; A.3A–C and A.3E; A.4A–C, with mathematical processes throughout. Advanced Grade 8 Algebra I, §111.31(d): (2)(A)–(G), (3)(A)–(F) and (H), and (4)(A)–(F). Codes from the two pathways are kept distinct.

These original Cambridge Veritas supplementary tutoring notes follow the agreed teaching sequence. They are not an official TEA publication. The class calendar is the Cambridge Veritas calendar, not a state pacing requirement.

Official references

Texas Education Agency: [Current mathematics standards index](#).

Standard Algebra I: [§111.39, high-school mathematics TEKS](#).

Advanced Grade 8 Algebra I: [§111.31, middle-school mathematics TEKS, adopted 2025](#).

Standards checked September 9, 2026. Chapter 2 edition: 1.0.