

LESSON 33 | MEET THE IDEA

Quadratic zeros, factoring and equations from roots

Tuesday, February 23, 2027 . 6:00–7:00 PM Central

By the end, I can...

Solve a factorable quadratic equation, connect its roots to graph intercepts, and build a quadratic from roots and a further condition.

Last chapter, factoring changed an expression into a product. Now an **equation** asks which x -values make the two sides equal. A **root** is a solution of the equation.

The zero-product property

If $AB = 0$, then $A = 0$ or $B = 0$ (or both).

For real numbers, two nonzero factors cannot have product zero.

Use this rule only when the product equals **zero**.

Solve $x^2 - 5x + 6 = 0$

$$(x - 2)(x - 3) = 0$$

Factor: -2 and -3 have product 6 and sum -5 .

$$x - 2 = 0 \text{ or } x - 3 = 0$$

Apply the zero-product property.

$$x = 2 \text{ or } x = 3$$

Solve each linear equation.

Check in the original equation: $2^2 - 5(2) + 6 = 0$ and $3^2 - 5(3) + 6 = 0$.

For the function $f(x) = x^2 - 5x + 6$, these same numbers are its **zeros**: $f(2) = f(3) = 0$. The x -intercepts are the points $(2, 0)$ and $(3, 0)$.

LESSON 33 | KEEP EVERY POSSIBLE SOLUTION

Set the equation equal to zero first

Solve $2x^2 - 7x = 4$

$$2x^2 - 7x - 4 = 0$$

Subtract 4 from both sides before factoring.

$$(2x + 1)(x - 4) = 0$$

The middle terms are $-8x + x = -7x$.

$$x = -1/2 \text{ or } x = 4$$

Solve $2x + 1 = 0$ and $x - 4 = 0$.

Check: at $x = -1/2$, $2x^2 - 7x = 1/2 + 7/2 = 4$. At $x = 4$, it is $32 - 28 = 4$.

Solve $x^2 = 3x$ without losing a root

$$x^2 - 3x = 0$$

Subtract $3x$ from both sides.

$$x(x - 3) = 0$$

Factor out x ; do not divide by x .

$$x = 0 \text{ or } x = 3$$

Both values satisfy the original equation.

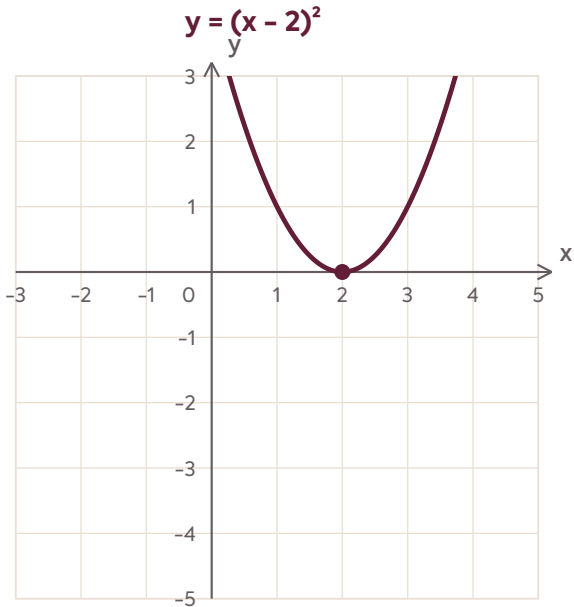
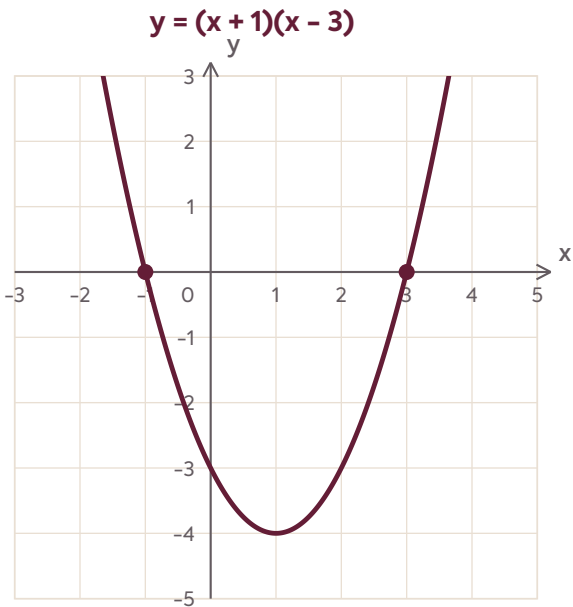
A repeated root counts as one distinct value

$$x^2 - 4x + 4 = (x - 2)^2 = 0 \text{ gives } x = 2.$$

The factor occurs twice, so 2 is a **repeated root**. There is one distinct real solution.

LESSON 33 | FACTORS, ZEROS AND INTERCEPTS

See the roots on the graph



Function	Real zeros	x-intercepts
$y = (x + 1)(x - 3)$	-1 and 3	$(-1, 0)$, $(3, 0)$
$y = (x - 2)^2$	2, a repeated zero	$(2, 0)$

The left parabola crosses the axis at each zero. The right parabola touches it at the repeated zero. The marked dots are intercepts; the curves continue beyond the graph windows.

Read the sign from the equation inside the factor

The factor $x + 1$ gives $x + 1 = 0$, so the zero is **-1**.

The factor $x - 3$ gives the zero **3**.

Do not copy the number inside the parentheses without solving.

LESSON 33 | A FURTHER POINT FIXES THE SCALE

Build a quadratic from its roots

For real roots r_1 and r_2 , write $y = a(x - r_1)(x - r_2)$, with $a \neq 0$. The roots tell you the factors. A leading coefficient or another suitable point determines a .

Roots -1 and 3; the graph passes through (0, -6)

$$y = a(x + 1)(x - 3)$$

Use one factor for each root.

$$-6 = a(0 + 1)(0 - 3)$$

Substitute the additional point.

$$-6 = -3a, \text{ so } a = 2$$

Divide by -3.

$$y = 2(x + 1)(x - 3)$$

This is the required function.

Standard form: $y = 2x^2 - 4x - 6$. At $x = 0$, $y = -6$; at $x = -1$ or 3 , $y = 0$.

Two roots alone do not choose one curve

$y = (x + 1)(x - 3)$ and $y = 2(x + 1)(x - 3)$ have the same zeros but different shapes. If a question asks for the **monic** quadratic, set $a = 1$. Otherwise retain a until another condition is given.

A repeated root r gives $y = a(x - r)^2$. To determine a from a second point, its x -coordinate must differ from the root; another intercept adds no new scale information.

LESSON 33 | SHORT CHECKS

Your turn

1. Solve $x^2 + x - 12 = 0$. Check both solutions.

2. Solve $2x^2 + 3x = 2$. Start by making one side zero.

3. For $y = (x + 2)^2$, state the zero, x-intercept and whether the root is repeated.

4. Write the monic quadratic function with roots -2 and 5 . Give factored and standard forms.

5. A graph has x-intercepts $(1, 0)$, $(4, 0)$ and passes through $(0, 8)$. Find its equation.

6. A student divides $x^2 = 5x$ by x and reports only $x = 5$. Explain the missing case.

LESSON 33 | WORKED ANSWERS

Check every root and the scale

1. $(x + 4)(x - 3) = 0$, so $x = -4$ or $x = 3$. Checks: $16 - 4 - 12 = 0$ and $9 + 3 - 12 = 0$.

2. $2x^2 + 3x - 2 = (2x - 1)(x + 2) = 0$.

$x = 1/2$ or $x = -2$. In the original equation: $1/2 + 3/2 = 2$ and $8 - 6 = 2$.

3. The zero is -2 ; the x -intercept is $(-2, 0)$. It is a **repeated root** because the factor $x + 2$ occurs twice.

4. Monic means $a = 1$: $y = (x + 2)(x - 5) = x^2 - 3x - 10$. Each given root makes a factor zero.

5. $y = a(x - 1)(x - 4)$. Using $(0, 8)$: $8 = 4a$, so $a = 2$.

$y = 2(x - 1)(x - 4) = 2x^2 - 10x + 8$.

6. Dividing by x assumes $x \neq 0$ and loses a valid solution. Instead, $x(x - 5) = 0$ gives $x = 0$ or $x = 5$. Both satisfy the original equation.

I can...

- ☐ Set an equation equal to zero before using a factored product.
- ☐ Connect a root to its x -intercept.
- ☐ Use a further condition to determine a quadratic's scale.

Next class: solve by isolating a square. Curriculum focus: A.8A, A.7B, A.6C.