

LESSON 31 | MEET THE IDEA

Transformations of the quadratic parent function

Tuesday, February 16, 2027 . 6:00–7:00 PM Central

By the end, I can...

Predict how a change inside or outside $f(x)$ transforms the parent parabola, and confirm the prediction using points.

For this lesson, $f(x) = x^2$. A transformation changes its graph. Read **where** the number appears before deciding what it does.

New rule	What changes	Example
$af(x)$	Outputs multiply by a .	$2f(x) = 2x^2$
$f(x) + d$	Outputs increase by d .	$f(x) + 2 = x^2 + 2$
$f(x - c)$	Inputs are shifted; the graph moves right by c when $c > 0$.	$f(x - 2) = (x - 2)^2$
$f(bx)$, $b \neq 0$	Horizontal distances scale by $1/ b $; a negative b also reflects across the y -axis.	$f(2x) = (2x)^2 = 4x^2$

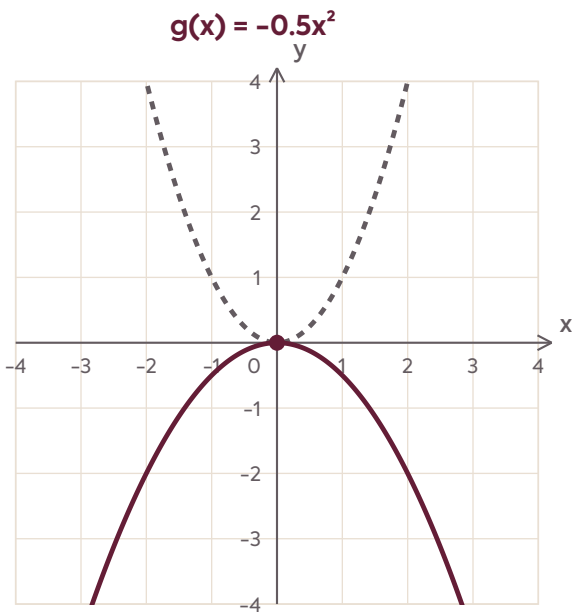
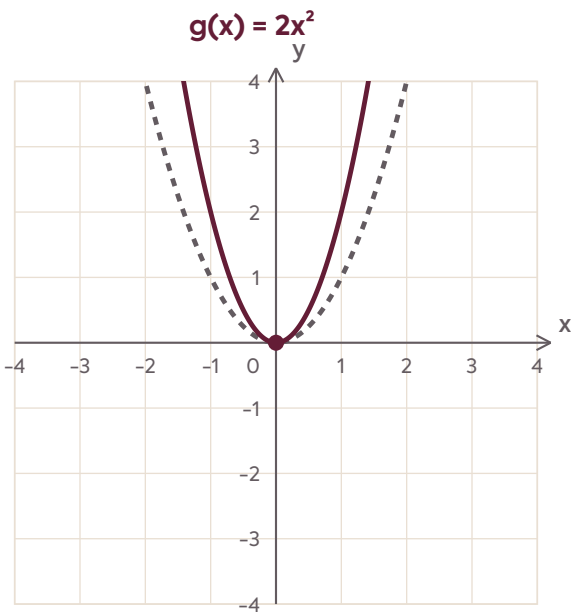
A point is a reliable check

Since $f(1) = 1$, the parent contains $(1, 1)$.
For $g(x) = 2f(x)$, that point becomes $(1, 2)$.
For $g(x) = f(2x)$, the corresponding point becomes $(1/2, 1)$.
The same number 2 appears in different places and does different jobs.

LESSON 31 | VERTICAL SCALING AND REFLECTION

Multiply outside: change the outputs

For $g(x) = af(x)$, a parent point (x, y) becomes **(x, ay)** . The x -coordinate stays the same.



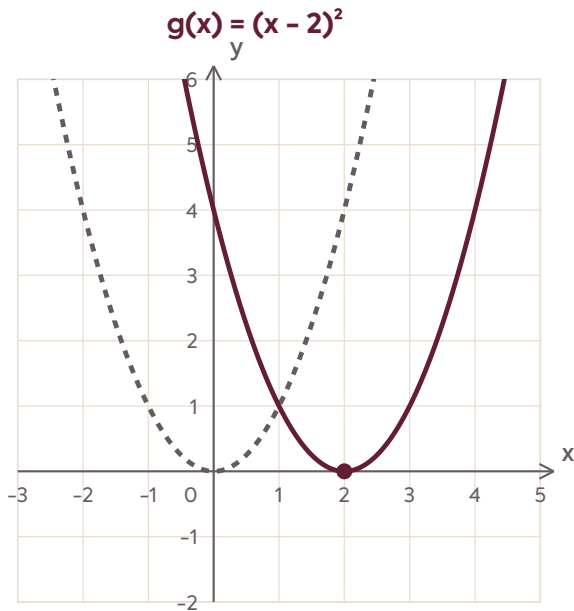
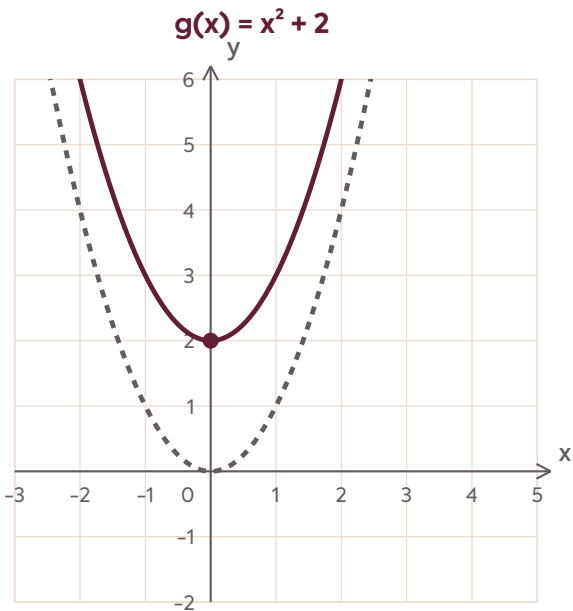
Dashed gray: $y = x^2$. **Solid burgundy:** the transformed graph. The same scale is used on both axes and in both panels.

Value of a	Effect	Point check
$a = 2$	Vertical stretch by 2; opens up.	$(1, 1)$ becomes $(1, 2)$.
$a = -1/2$	Vertical compression by $1/2$ and reflection across the x -axis; opens down.	$(2, 4)$ becomes $(2, -2)$.

For $|a| > 1$, the parabola looks narrower on the same axes. For $0 < |a| < 1$, it looks wider. The sign of a controls the opening. If $a = 0$, the result is constant, not quadratic.

LESSON 31 | VERTICAL AND HORIZONTAL SHIFTS

Add outside; subtract inside



Dashed gray: the parent. Solid burgundy: the transformed graph.

Rule	Movement	Vertex
$f(x) + 2 = x^2 + 2$	Up 2	(0, 2)
$f(x - 2) = (x - 2)^2$	Right 2	(2, 0)
$f(x) - 1.5 = x^2 - 1.5$	Down 1.5	(0, -1.5)
$f(x + 0.5) = (x + 0.5)^2$	Left 0.5	(-0.5, 0)

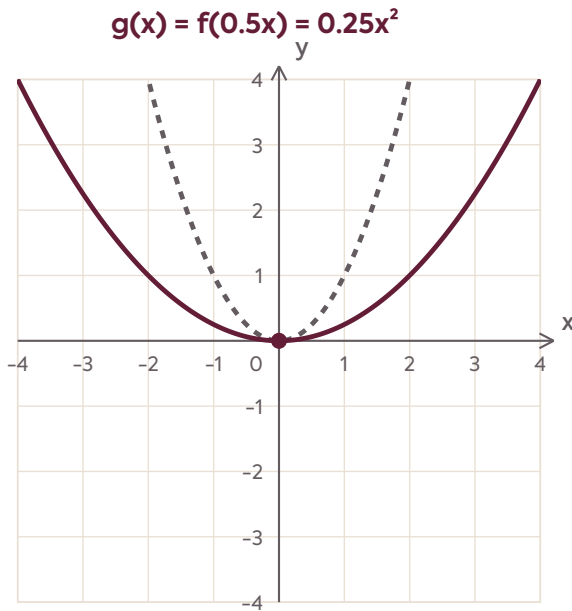
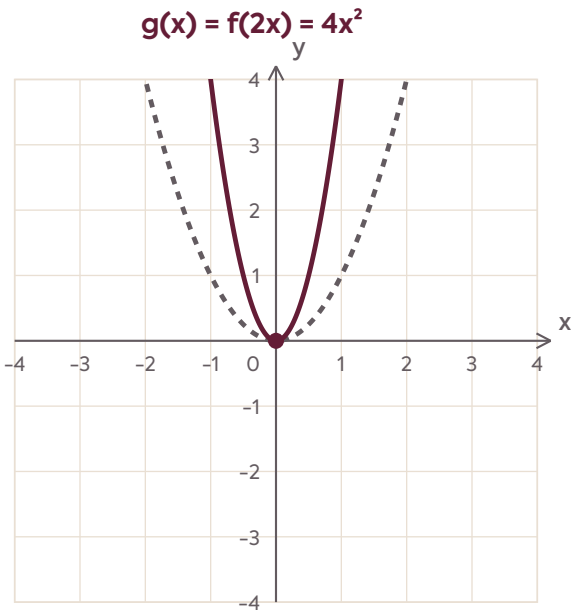
Why $x - 2$ shifts right

The parent has its lowest output when its input is 0. For $(x - 2)^2$, the inside becomes 0 at $x = 2$. That puts the vertex at (2, 0).

LESSON 31 | HORIZONTAL SCALING

Multiply inside: change the inputs

For $g(x) = f(bx)$, $b \neq 0$, a parent point (x, y) becomes $(x/b, y)$. The new input must make bx equal the old input.



Dashed gray: the parent. **Solid burgundy:** the transformed graph.

b	Horizontal change	Check with parent point (2, 4)
2	Compress distances by 1/2.	(1, 4) lies on $f(2x)$.
1/2	Stretch distances by 2.	(4, 4) lies on $f(x/2)$.

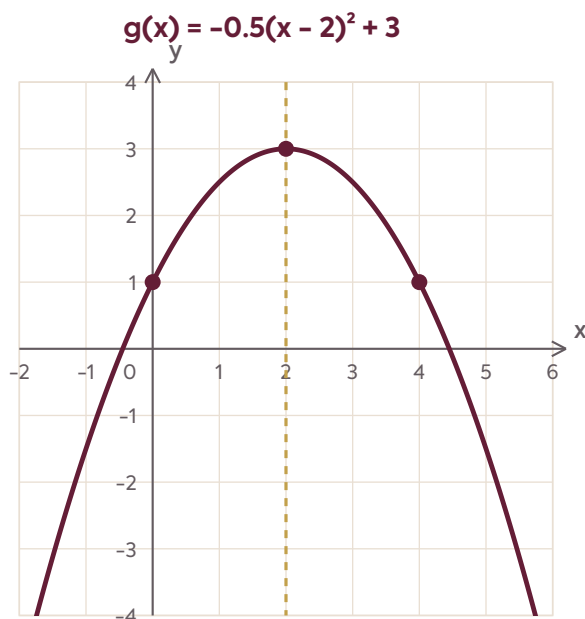
Negative input scaling does not open this parabola down

$f(-2x) = (-2x)^2 = 4x^2 = f(2x)$.
Reflecting $y = x^2$ across the y-axis leaves the same shape because the parent is symmetric. Also, $f(-x/2) = x^2/4$. A negative multiplier **outside** f would open it down.

LESSON 31 | READ VERTEX FORM

Put the changes together

In $g(x) = a(x - h)^2 + k$, the vertex is (h, k) . A parent point (u, u^2) becomes $(u + h, au^2 + k)$. This point rule keeps combined changes clear.



Vertex: $(2, 3)$

Axis: $x = 2$

Opening: downward

Maximum: 3

Range: $y \leq 3$

The points $(0, 1)$ and $(4, 1)$ are equally far from the axis.

Use the point rule for $g(x) = -0.5(x - 2)^2 + 3$

$(0, 0)$ becomes $(2, 3)$

Add 2 to the input; multiply the output by -0.5 , then add 3.

$(2, 4)$ becomes $(4, 1)$

$2 + 2 = 4$ and $-0.5(4) + 3 = 1$.

$(-2, 4)$ becomes $(0, 1)$

The equal outputs confirm symmetry around $x = 2$.

Here the parent is vertically compressed, reflected across the x -axis, shifted right 2 and shifted up 3. All real x are allowed. The maximum value is 3.

LESSON 31 | SHORT CHECKS

Your turn

Throughout, $f(x) = x^2$. Explain each answer using the formula or a point, not just how the curve looks.

1. For $g(x) = -3f(x)$, describe the changes and map the parent point (1, 1).

2. For $g(x) = f(x) - 2.5$, give the vertex and range.

3. For $g(x) = f(x + 3)$, give the horizontal shift and vertex.

4. Simplify $f(3x)$. Describe its horizontal scaling and map the parent point (3, 9).

5. Simplify $f(-x/2)$. Does it open up or down? Map (2, 4) using the input rule.

6. For $g(x) = 2(x + 1)^2 - 4$, give the vertex, axis, opening and range.

7. Are $2f(x)$ and $f(2x)$ the same function? Show a value that decides.

LESSON 31 | WORKED ANSWERS

Check inside versus outside

1. **Vertical stretch by 3 and reflection across the x-axis.** It opens down. The point (1, 1) becomes **(1, -3)**.
2. It shifts down 2.5. Vertex: **(0, -2.5)**; range: **$y \geq -2.5$** .
3. **Left 3**, with vertex **(-3, 0)**. The inside $x + 3$ equals 0 when $x = -3$.
4. $f(3x) = (3x)^2 = 9x^2$. Horizontal distances compress by **$1/3$** . The point (3, 9) becomes **(1, 9)**.
5. $f(-x/2) = (-x/2)^2 = x^2/4$, so it opens **up**. With $b = -1/2$, (2, 4) maps to $(2 \div (-1/2), 4) = \mathbf{(-4, 4)}$. The graph is wider than the parent.
6. Write $x + 1$ as $x - (-1)$. Vertex: **(-1, -4)**; axis: **$x = -1$** ; opening: **upward**; range: **$y \geq -4$** .
7. **No.** $2f(x) = 2x^2$, while $f(2x) = 4x^2$. At $x = 1$, the outputs are **2** and **4**.

I can...

- ☐ Locate a parameter inside or outside the function.
- ☐ Confirm a transformation by mapping a point.
- ☐ Distinguish a negative input scale from an output reflection.

Next class: build a quadratic from a vertex and a point. Curriculum focus: A.7C.