

LESSON 40 | MEET THE IDEA

Geometric sequences and multiplicative change

Thursday, March 25, 2027 . 6:00–7:00 PM Central

By the end, I can...

Identify a common ratio, write recursive and explicit geometric rules, and distinguish multiplication from repeated addition.

An **arithmetic** sequence adds a fixed amount. A **geometric** sequence multiplies by a fixed number called the **common ratio** r . We again use $n = 1, 2, 3, \dots$.

Term number n	1	2	3	4
Term value a_n	3	6	12	24

Divide the new term by the previous term

$$6 \div 3 = 2, 12 \div 6 = 2 \text{ and } 24 \div 12 = 2.$$

The common ratio is $r = 2$. Each step doubles the value.

The differences 3, 6 and 12 are not constant.

Check several consecutive ratios. For this method, the previous term must be nonzero: division by zero is undefined. Our ratio examples use nonzero first terms and nonzero ratios.

Multiplying by a fraction is still multiplying

80, 40, 20, 10, ... has $r = 1/2$. Each term is half the previous term. A geometric sequence does not have to increase.

LESSON 40 | GEOMETRIC RULES

Count the multiplication steps

Recursive rule

$$a_1 = 3; a_n = 2a_{n-1} \text{ for } n = 2, 3, \dots$$

Start with 3, then multiply the previous term by 2.

Explicit rule for 3, 6, 12, 24, ...

$$a_1 = 3 \times 2^0$$

At term 1, there have been zero multiplication steps.

$$a_2 = 3 \times 2^1$$

At term 2, there has been one step.

$$a_n = 3 \times 2^{n-1}$$

For $n = 1, 2, \dots$, there are $n - 1$ steps.

$$a_6 = 3 \times 2^5 = 96$$

Use the fifth power for the sixth term.

General geometric rules

Explicit: $a_n = a_1 r^{n-1}$, $n = 1, 2, \dots$

Recursive: give a_1 , then $a_n = r a_{n-1}$ for $n = 2, 3, \dots$

The first index and first value must be stated.

An exponent counts factors, not an amount to add. For example, 3×2^5 means 3×32 , not $3 \times 2 \times 5$.

LESSON 40 | TWO USEFUL CASES

Fractions shrink; negative ratios alternate

160, 80, 40, 20, ...

$$r = 80/160 = 1/2$$

Each term keeps half of the previous value.

$$a_n = 160(1/2)^{n-1}$$

The first term is $n = 1$.

$$a_7 = 160/64 = 2.5$$

Six halvings occur between terms 1 and 7.

2, -6, 18, -54, ...

$$r = -6/2 = -3$$

$18/(-6)$ and $-54/18$ are also -3 .

$$a_n = 2(-3)^{n-1}$$

Parentheses keep the negative sign inside the base.

$$a_5 = 2(-3)^4 = 162$$

An even power of a negative base is positive.

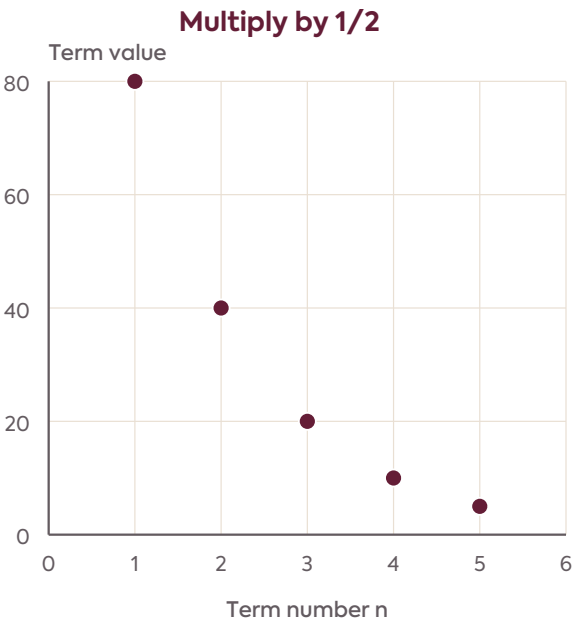
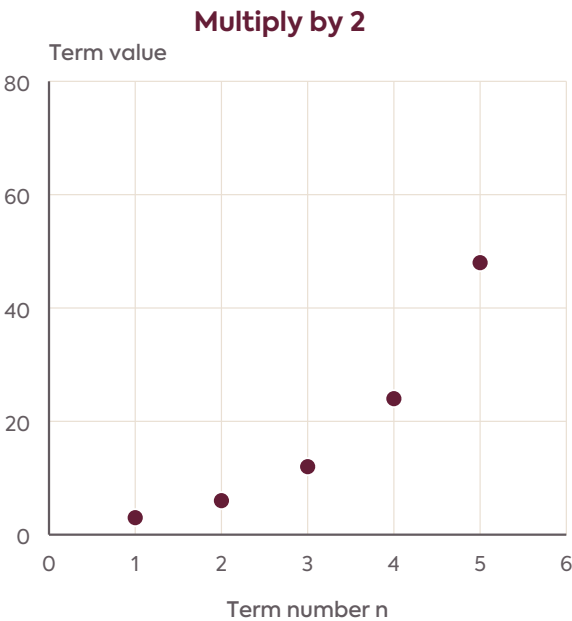
Do not turn a negative ratio into a continuous curve

The rule $2(-3)^{n-1}$ works for integer term numbers. It is **not** a standard real exponential function on every real input: a negative base does not give a real value for every fractional exponent.

For a positive starting value: $r > 1$ increases values; $0 < r < 1$ decreases them toward zero; $r = 1$ keeps them constant. A negative ratio alternates signs.

LESSON 40 | COMPARE THE PATTERNS

Look for equal ratios, not a straight line



Both panels use the same scales. Each dot is a term; no fractional indices are included. The doubling pattern bends upward, while the halving pattern gets closer to zero.

Sequence (n begins at 1)	Test	Explicit rule
3, 6, 9, 12, ...	Difference always 3: arithmetic	$a_n = 3 + 3(n - 1) = 3n$
3, 6, 12, 24, ...	Ratio always 2: geometric	$a_n = 3 \times 2^{n-1}$
2, 4, 7, 11, ...	Neither differences nor ratios stay constant	Neither pattern applies

A number pattern may be neither - or both

Do not force every list into one type. A nonzero constant sequence, such as 5, 5, 5, ... , is both arithmetic ($d = 0$) and geometric ($r = 1$).

LESSON 40 | SHORT CHECKS

Your turn

First index: $n = 1$. Assume each pattern continues as stated. Include the first term and allowed indices when writing a recursive rule.

1. For 5, 15, 45, ... , write both rules and find a_5 .

2. For 96, 48, 24, ... , find r , an explicit rule and a_6 .

3. $a_1 = 4$; $a_n = -2a_{n-1}$ for $n = 2, 3, \dots$. List the first four terms and give an explicit rule.

4. Classify each: (a) 4, 9, 14, 19, ... ; (b) 4, 8, 16, 32, ... ; (c) 1, 2, 4, 7, Explain your tests.

5. A student uses $a_n = 7 \times 3^n$ for 7, 21, 63, ... with first index 1. Correct the rule and explain.

6. Can $2(-3)^{n-1}$ be graphed as a standard real exponential curve for every real n ? Why?

LESSON 40 | WORKED ANSWERS

Check the factor and the exponent

1. $r = 3$. Recursive: $a_1 = 5$; $a_n = 3a_{n-1}$, $n = 2, 3, \dots$. Explicit: $a_n = 5 \times 3^{n-1}$, $n = 1, 2, \dots$. $a_5 = 5 \times 81 = 405$.

2. $r = 1/2$. The rule is $a_n = 96(1/2)^{n-1}$, $n = 1, 2, \dots$. $a_6 = 96/32 = 3$.

3. The terms are 4, -8, 16, -32. Explicit: $a_n = 4(-2)^{n-1}$, $n = 1, 2, \dots$. The signs alternate because the common ratio is negative.

4. (a) **Arithmetic**: differences are 5. (b) **Geometric**: ratios are 2. (c) **Neither**: differences 1, 2, 3 are not constant, and ratios 2, 2, 7/4 are not constant.

5. Use $a_n = 7 \times 3^{n-1}$, $n = 1, 2, \dots$. The first term uses exponent zero: $7 \times 1 = 7$. The proposed rule gives 21 at $n = 1$ and starts one step too far ahead.

6. **No**. The integer-indexed sequence is valid. But, for example, $n = 1.5$ would require the square root of -3, which is not real. Standard real exponential functions use a positive base.

I can...

- ☐ Distinguish a common ratio from a common difference.
- ☐ Use $n - 1$ when the first term has index 1.
- ☐ Handle fractional and negative ratios carefully.

Next class: build exponential growth and decay models. Curriculum focus: A.12C-D; review A.11B.