

LESSON 21 | MEET THE IDEA

Integer exponent laws

Tuesday, January 12, 2027 . 6:00–7:00 PM Central

By the end, I can...

Use integer exponent laws, explain zero and negative exponents, and keep the restrictions that make an expression valid.

An **exponent** tells how many copies of the base are multiplied when the exponent is a positive integer. The base and exponent have different jobs.

$$2^3 = 2 \times 2 \times 2 = 8$$

base 2 exponent 3

Three factors of 2, not 2×3 .

Why we add exponents when multiplying the same base

$$x^3 \times x^2$$

Write the factors: $(x \times x \times x)(x \times x)$.

$$x^{3+2} = x^5$$

There are five factors of x altogether.

Product rule

$$a^m \times a^n = a^{m+n}$$

The bases must match. For example, $2^3 \times 3^2$ cannot become 6^5 . Work out $8 \times 9 = 72$ instead.

LESSON 21 | THREE MORE RULES

Divide powers and raise a power

Quotient rule | x^5 / x^2 , with $x \neq 0$

$(x \times x \times x \times x \times x)$
 $/ (x \times x)$
 $x^{5-2} = x^3$

Cancel two nonzero factors from top and bottom.

Subtract the denominator exponent from the numerator exponent.

Rule	Example
$a^m / a^n = a^{m-n}$, $a \neq 0$	$a^7 / a^3 = a^4$
$(a^m)^n = a^{mn}$	$(x^3)^2 = x^6$
$(ab)^n = a^n b^n$	$(2x^3)^2 = 4x^6$
$(a/b)^n = a^n / b^n$, $b \neq 0$	$(3x/y)^2 = 9x^2/y^2$

For positive integer powers, these rules apply wherever the original expression is defined. With zero or negative exponents, nonzero-base restrictions also matter; the next page explains them.

Product or power of a power?

$x^3 \times x^2 = x^5$: add $3 + 2$.
 $(x^3)^2 = x^6$: multiply 3×2 .
Read the operation before choosing a rule.

LESSON 21 | EXTEND A PATTERN

Zero and negative exponents

As the exponent drops by 1, divide the value by 2. Keep that pattern going through zero and below.

Power	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}
Value	8	4	2	1	1/2	1/4

Why $a^0 = 1$ when $a \neq 0$

$$a^3 / a^3 = 1$$

A nonzero quantity divided by itself is 1.

$$a^{3-3} = a^0$$

The quotient rule gives the same value.

A negative exponent means a reciprocal

For $a \neq 0$ and positive integer n :

$$a^{-n} = 1/a^n.$$

Examples: $5^{-2} = 1/25$; $x^{-3} = 1/x^3$, $x \neq 0$. It does **not** mean $-a^n$.

Keep parentheses around a negative base

$(-2)^4 = 16$: four negative factors multiply to a positive number.

$-2^4 = -16$: raise 2 to the fourth power, then apply the minus sign.

Zero raised to a positive integer power is 0. Zero to a negative power would require division by 0, so it is undefined. In this course, we also leave 0^0 undefined.

LESSON 21 | KEEP TRACK OF EVERY FACTOR

Combine the rules carefully

Simplify $(6x^5y^{-2}) / (3x^2y)$, with $x \neq 0$ and $y \neq 0$

$$2x^{5-2}y^{-2-1}$$

Divide coefficients; subtract exponents for matching bases.

$$2x^3y^{-3}$$

Simplify the exponent calculations.

$$2x^3 / y^3$$

Rewrite with positive exponents.

Keep $x \neq 0$ and $y \neq 0$ from the original expression, even though the final expression no longer has x in its denominator.

A powers-of-ten example

$$(1.2 \times 10^3)(3 \times 10^2)$$

A model has 1,200 groups, each with 300 objects.

$$(1.2 \times 3) \times 10^{3+2}$$

Multiply the coefficients and add the powers of 10.

$$3.6 \times 10^5 = 360,000$$

The model contains 360,000 objects.

Do not use a multiplication rule for addition

$$x^2 + x^2 = 2x^2, \text{ not } x^4.$$

$$x^2 \times x^2 = x^4.$$

The operation sign tells you what to do.

LESSON 21 | SHORT CHECKS

Your turn

Try these before the worked answers. Write positive exponents in final answers and keep the stated restrictions.

1. Simplify $x^4 \times x^3$.

2. Simplify a^8 / a^3 , with $a \neq 0$.

3. Simplify $(3p^2q^3)^2$.

4. Rewrite $4x^{-2}$, with $x \neq 0$, using a positive exponent.

5. Simplify $(8a^5b^{-1}) / (2a^2b^2)$, with $a \neq 0$ and $b \neq 0$.

6. Evaluate $(-3)^2$ and -3^2 . Explain why the signs differ.

7. Calculate $(2 \times 10^3)(4 \times 10^2)$. Give a power-of-ten form and an ordinary number.

LESSON 21 | WORKED ANSWERS

Check the rule you chose

1. $x^{4+3} = x^7$. Same base, multiplication: add the exponents.
2. $a^{8-3} = a^5$, keeping $a \neq 0$ from the original denominator.
3. $3^2(p^2)^2(q^3)^2 = 9p^4q^6$. Square every factor.
4. $4/x^2$, $x \neq 0$. Only x has exponent -2 ; the coefficient 4 is not squared or inverted.
5. $4a^{5-2}b^{-1-2} = 4a^3b^{-3} = 4a^3/b^3$, with $a \neq 0$ and $b \neq 0$.
6. $(-3)^2 = 9$, because $(-3)(-3) = 9$. $-3^2 = -9$, because the exponent applies to 3 before the minus sign.
7. $(2 \times 4) \times 10^{3+2} = 8 \times 10^5 = 800,000$.

A useful self-check

Try a simple allowed value such as $x = 2$ in your original and simplified expressions. Different values expose an error. One matching test supports your work but does not prove two expressions are always equivalent.

I can...

- ☐ Distinguish multiplication from a power of a power.
- ☐ Interpret zero and negative exponents.
- ☐ Keep nonzero restrictions from the original expression.

Next class: square roots and fractional exponents. Curriculum focus: A.11B.