

LESSON 41 | MEET THE IDEA

Writing exponential growth and decay models

Tuesday, March 30, 2027 . 6:00–7:00 PM Central

By the end, I can...

Build an exponential model from an initial value and a repeated percentage change, and explain what each parameter means.

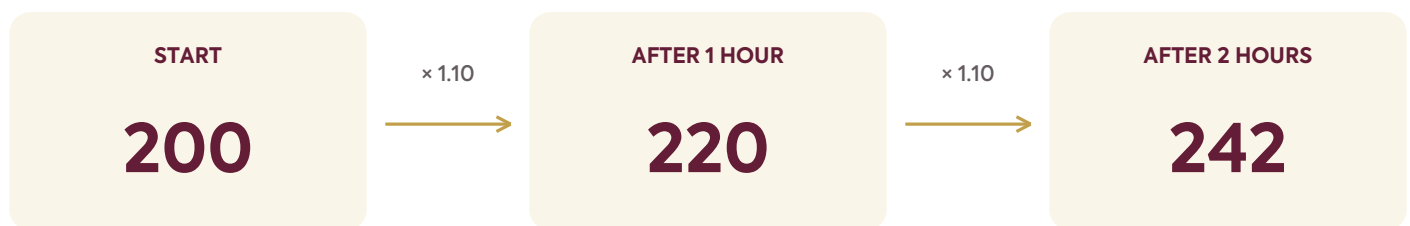
An initial amount times a repeated multiplier

$$f(x) = ab^x$$

a is the output at $x = 0$; **b** is the factor for each increase of 1 in x .

For our nonconstant real exponential models, $b > 0$, $b \neq 1$ and $a \neq 0$.

For positive quantities we use $a > 0$. Growth has $b > 1$. Decay has $0 < b < 1$. The exponent counts equal time periods, not the amount added.



A model starts at 200 and grows 10% per hour.

$$b = 1 + 0.10 = 1.10$$

Keep the original 100%, then add 10%.

$$f(t) = 200(1.10)^t$$

t is hours since the start, with $t = 0$ initially.

$$f(2) = 200(1.10)^2 = 242$$

The second 10% increase is based on 220, not 200.

LESSON 41 | WHAT STAYS AFTER ONE PERIOD?

Turn a percentage into a multiplier

Change per period	Decimal rate <i>r</i>	Multiplier <i>b</i>
Increase by 8%	0.08	$1 + 0.08 = 1.08$
Decrease by 8%	0.08	$1 - 0.08 = 0.92$
Increase by 25%	0.25	1.25
Decrease by 25%	0.25	0.75

A fictional display fades by 20% each minute from intensity 500.

$b = 1 - 0.20 = 0.80$

A 20% loss leaves 80%, not 20%.

$I(t) = 500(0.80)^t$

t is the number of minutes since fading began.

$I(3) = 500(0.80)^3 = 256$

After three minutes, the model predicts 256 intensity units.

Reverse the conversion

For a positive quantity: ***b* = 1.06** means a 6% increase.

***b* = 0.94** means a 6% decrease.

If $b > 1$, growth rate = $b - 1$. If $0 < b < 1$, decay rate = $1 - b$. Multiply the decimal rate by 100 to express it as a percent.

LESSON 41 | USE EQUAL INPUT STEPS

Write the model from a table

Time t (hours)	0	1	2	3
Output	80	100	125	156.25

The table follows an exact exponential rule.

- $a = 80$

$b = 100/80 = 1.25$

$f(t) = 80(1.25)^t$
- Read the output at $t = 0$.

Inputs are one hour apart. Check $125/100$ and $156.25/125$ too.

The model grows by 25% per hour.

What if $t = 0$ is not listed?

- $f(1) = 60; f(2) = 90$

$b = 90/60 = 1.5$

$a = 60/1.5 = 40$

$f(t) = 40(1.5)^t$
- Assume a positive-base exponential model.

The two input values differ by 1.

Step backward once to $t = 0$.

Check $t = 1$ gives 60 and $t = 2$ gives 90.

Real data usually have imperfect ratios; we will fit such data in Class 43. A model assumption matters: a few points alone do not prove a unique rule for all inputs.

LESSON 41 | TIME PERIODS AND SEQUENCE TERMS

Keep the clock and the index clear

A quantity starts at 40 and doubles every 3 hours.

Number of doubling periods =
 $t/3$

$P(t) = 40 \times 2^{t/3}$

$P(6) = 40 \times 2^2 = 160$

t is measured in hours.

One doubling occurs at $t = 3$, two at $t = 6$.

40×2^6 would incorrectly double every hour.

Meaning	Input convention	Rule
Amount after t hours	$t = 0$ is the start	$f(t) = 40 \times 2^{t/3}$
Readings every 3 hours, starting now	Reading 1 is at $t = 0$	$a_n = 40 \times 2^{n-1}$, $n = 1, 2, \dots$

Choose continuous or discrete inputs from the story

If the process is modeled throughout time, allow real $t \geq 0$.

If only completed hourly rounds count, use $t = 0, 1, 2, \dots$.

A sequence of readings has integer indices even when the underlying process is continuous.

The hourly multiplier in the doubling example is $2^{1/3} \approx 1.260$. Thus $40(2^{1/3})^t$ is equivalent to $40 \times 2^{t/3}$. Units determine which multiplier to use.

LESSON 41 | SHORT CHECKS

Your turn

1. A model begins at 120 and grows 5% per day. Write $f(t)$ and find $f(2)$, where t is days since the start.

2. A signal starts at 800 units and decreases 15% per hour. Write $S(t)$ and find $S(2)$.

3. For $f(t) = 60(0.7)^t$, explain 60 and 0.7. What is the percent decrease per time period?

4. An exact exponential table has $(t, y) = (0, 12), (1, 18), (2, 27)$. Find the rule and percentage growth.

5. A quantity starts at 10 and triples every 2 hours. Write a model using t in hours and find its value at $t = 4$.

6. Readings begin at time 0 for $f(t) = 5(1.2)^t$ and are taken hourly. Give the sequence rule when the first reading is $n = 1$.

LESSON 41 | WORKED ANSWERS

Check the amount retained

1. $f(t) = 120(1.05)^t$, with t in days since the start. $f(2) = 120 \times 1.1025 = \mathbf{132.3}$. The multiplier includes the original 100% plus 5%.
2. A 15% decrease leaves 85%: $S(t) = 800(0.85)^t$. $S(2) = 800 \times 0.7225 = \mathbf{578 \text{ units}}$.
3. **60** is the initial output at $t = 0$. **0.7** means 70% remains after each period, a **30% decrease** because $1 - 0.7 = 0.3$.
4. $a = 12$ and $b = 18/12 = 27/18 = 1.5$. The rule is $f(t) = 12(1.5)^t$; it grows by **50%** per one-unit increase in t .
5. $P(t) = 10 \times 3^{t/2}$. Four hours contains two tripling periods: $P(4) = 10 \times 9 = \mathbf{90}$.
6. Reading n occurs at $t = n - 1$. Therefore $a_n = 5(1.2)^{n-1}$, $n = 1, 2, \dots$. The first reading is 5, not 6.

I can...

- ☐ Translate a percentage increase or decrease into a multiplier.
- ☐ State the units and meaning of the input.
- ☐ Distinguish time 0 from sequence term 1.

Next class: graph these models and describe their domains and ranges. Curriculum focus: A.9B–C; review A.11B and A.12D.