

LESSON 17 | MEET THE IDEA

Solving systems by substitution

Tuesday, December 15, 2026 . 6:00–7:00 PM Central

By the end, I can...

Replace a variable with an equal expression, solve the resulting equation and check the ordered pair in both original equations.

Substitution means replacing something with an equal value or expression. If $y = 2x + 1$, then $2x + 1$ can take the place of y in the other equation.

Example 1 | $y = 2x + 1$ and $3x + y = 11$

$$3x + (2x + 1) = 11$$

Replace y in the second equation by its equal expression.

$$5x + 1 = 11$$

Combine the x terms.

$$5x = 10, \text{ so } x = 2$$

Subtract 1, then divide by 5.

$$y = 2(2) + 1 = 5$$

Use $x = 2$ to find the other coordinate.

The solution is $(2, 5)$. Check: $5 = 2(2) + 1$ and $3(2) + 5 = 11$.

Choose the easiest entry point

Look for a variable already alone, or one with coefficient 1 or -1 that is easy to isolate. Either variable can work. Keep the first equation available so you can find the missing coordinate.

LESSON 17 | PARENTHESES PROTECT THE MEANING

Replace the whole expression

Example 2 | $x = 2y - 1$ and $3x - 2y = 9$

$$3(2y - 1) - 2y = 9$$

Replace the entire x with $(2y - 1)$.

$$6y - 3 - 2y = 9$$

Distribute 3 to both terms.

$$4y - 3 = 9$$

Combine like terms.

$$4y = 12, \text{ so } y = 3$$

Add 3, then divide by 4.

$$x = 2(3) - 1 = 5$$

Substitute back to find x .

Solution: $(5, 3)$. Checks: $5 = 2(3) - 1$, and $3(5) - 2(3) = 15 - 6 = 9$.

Spot the error

Incorrect: $3(2y - 1) = 6y - 1$.

Correct: $3(2y - 1) = 6y - 3$.

The multiplier acts on every term inside the parentheses.

What if neither variable is alone?

For $x + y = 10$, subtract x to get $y = 10 - x$. Now substitute $10 - x$ wherever y appears in the other equation. Rewriting an equation first is part of the method.

LESSON 17 | TICKET SALES

Make the answer fit the story

A school sells 18 tickets. Adult tickets cost \$8 and student tickets cost \$5. Sales total \$108. Let a be adult tickets and s be student tickets; both must be nonnegative whole numbers.

Relationship	Equation
18 tickets in total	$a + s = 18$
\$108 in sales	$8a + 5s = 108$

Use the ticket count to isolate s

$s = 18 - a$

$8a + 5(18 - a) = 108$

$8a + 90 - 5a = 108$

$3a = 18$, so $a = 6$

$s = 18 - 6 = 12$

Solve the simpler equation for s .

Replace s in the money equation.

Distribute 5, including its effect on $-a$.

Combine like terms, subtract 90 and divide by 3.

Find the number of student tickets.

Answer and check

6 adult tickets and 12 student tickets.

Count: $6 + 12 = 18$.

Sales: $8(6) + 5(12) = 48 + 60 = \108 .

Both counts are whole numbers and satisfy both conditions.

LESSON 17 | TWO SPECIAL OUTCOMES

When the variables disappear

Substitution does not always leave an equation with one numerical answer. Sometimes the variables cancel. Read what remains.

Same line | $y = 2x + 1$ and $2y = 4x + 2$

$$2(2x + 1) = 4x + 2$$

Substitute for y .

$$4x + 2 = 4x + 2$$

Both sides are the same expression.

$$2 = 2$$

A true statement remains for every x .

There are infinitely many solutions: all points on $y = 2x + 1$. It does not mean every point in the plane is a solution.

Parallel lines | $y = 2x + 1$ and $2y = 4x + 6$

$$2(2x + 1) = 4x + 6$$

Substitute for y .

$$4x + 2 = 4x + 6$$

Distribute.

$$2 = 6$$

A false statement remains. No x can fix it.

There is no solution: the two original equations cannot be true together.

Do not turn $0 = 0$ into $x = 0$

An identity such as $0 = 0$ places no new restriction on x . Use the original equation to describe the shared line. A contradiction such as $0 = 4$ means no shared point.

LESSON 17 | SHORT CHECKS

Your turn

Show your substitution with parentheses, give an ordered pair and check both original equations.

1. Solve $y = x + 2$ and $x + y = 10$.

2. Solve $x = 3y - 2$ and $2x + y = 17$.

3. There are 15 tickets: adult tickets cost \$8, student tickets cost \$5, and sales total \$96. Define variables, write the system and solve.

4. How many solutions does $y = 3x - 2$ with $2y = 6x - 4$ have? Use substitution to explain.

5. For $y = x + 4$ and $3x + 2y = 18$, a student writes $3x + 2x + 4 = 18$. Correct the substitution and solve.

LESSON 17 | WORKED ANSWERS

Check your substitutions

1. $x + (x + 2) = 10$ gives $2x = 8$, so $x = 4$; then $y = 6$. **(4, 6)**. Checks: $6 = 4 + 2$ and $4 + 6 = 10$.
2. $2(3y - 2) + y = 17$ gives $7y - 4 = 17$, so $y = 3$; $x = 7$. **(7, 3)**. Checks: $7 = 3(3) - 2$ and $2(7) + 3 = 17$.
3. Let a and s count adult and student tickets. $a + s = 15$ and $8a + 5s = 96$. Substitute $s = 15 - a$: $8a + 5(15 - a) = 96$, so $3a = 21$. **7 adult tickets and 8 student tickets**. Checks: $7 + 8 = 15$ and $8(7) + 5(8) = 96$.
4. $2(3x - 2) = 6x - 4$ gives $6x - 4 = 6x - 4$, a true statement for every x . **Infinitely many solutions, all on $y = 3x - 2$** .
5. Use **$3x + 2(x + 4) = 18$** . Then $5x + 8 = 18$, so $x = 2$ and $y = 6$. **(2, 6)**. Checks: $6 = 2 + 4$ and $3(2) + 2(6) = 18$. The student failed to multiply 4 by 2.

One equation is not enough for the final check

Back-substitution finds the second coordinate, but only checking both original equations confirms that your pair solves the system.

I can...

- ☐ Choose a useful variable to isolate.
- ☐ Substitute, solve and check both equations.
- ☐ Recognize an identity or contradiction.

Next class: elimination combines equations to remove one variable. Curriculum focus: A.5C.