

LESSON 35 | MEET THE IDEA

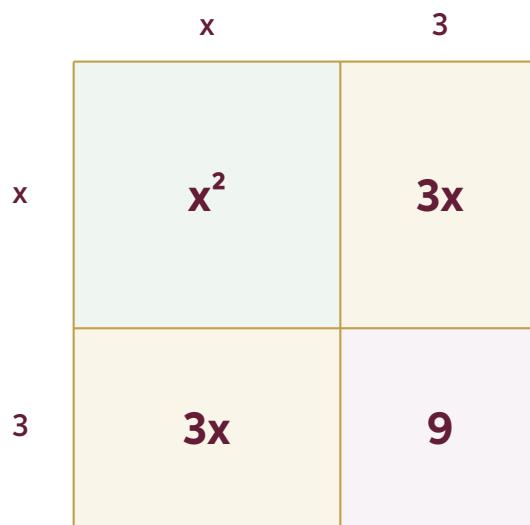
Solving quadratics by completing the square

Tuesday, March 2, 2027 . 6:00–7:00 PM Central

By the end, I can...

Create a perfect-square trinomial without changing an equation's solutions, then solve it by taking square roots.

To **complete the square**, add the missing term that makes a quadratic expression a squared binomial. The area picture explains why we halve the x-coefficient and square the result.



Two strips of area $3x$ make $6x$. The missing corner is $3 \times 3 = 9$.

$$x^2 + 6x + 9 = (x + 3)^2$$

Area illustration uses $x = 4$; the algebraic identity holds for all real x .

Half, then square

For $x^2 + bx$, add $(b/2)^2$ to make $(x + b/2)^2$.

Example: $b = 6$, half is 3, and $3^2 = 9$.

In an **equation**, add the same quantity to both sides.

LESSON 35 | A COMPLETE WORKED EXAMPLE

Keep the equation balanced

Solve $x^2 + 6x = 7$

Half of 6 is 3; $3^2 = 9$

$x^2 + 6x + 9 = 7 + 9$

$(x + 3)^2 = 16$

$x + 3 = \pm 4$

$x = 1$ or $x = -7$

Check: $1^2 + 6(1) = 7$ and $(-7)^2 + 6(-7) = 49 - 42 = 7$.

Find the term needed to complete the square.

Add 9 to both sides, preserving equality.

Rewrite the left as a perfect square.

Use both square roots of 16.

Subtract 3 from each possibility.

Expression	Half the x-coefficient	Term to add	Squared form
$x^2 + 8x$	4	16	$(x + 4)^2$
$x^2 - 4x$	-2	4	$(x - 2)^2$
$x^2 + 5x$	5/2	25/4	$(x + 5/2)^2$

A negative half-coefficient still gives a positive added square. A fractional half-coefficient is allowed: $(5/2)^2 = 25/4$.

LESSON 35 | A SIMPLE NONMONIC EQUATION

Make the leading coefficient 1 first

Solve $2x^2 + 8x - 6 = 0$

$$x^2 + 4x - 3 = 0$$

$$x^2 + 4x = 3$$

$$x^2 + 4x + 4 = 3 + 4$$

$$(x + 2)^2 = 7$$

$$x = -2 \pm \sqrt{7}$$

Divide EVERY term on both sides by 2.

Move the constant to the other side.

Half of 4 is 2; add $2^2 = 4$ to both sides.

The left side is now a square.

Take square roots, then subtract 2.

Check using an equivalent form

$$2x^2 + 8x - 6 = 2[(x + 2)^2 - 7].$$

For either root, $(x + 2)^2 = 7$, so the original expression becomes $2(7 - 7) = 0$.

Do not halve the original coefficient 8 while leaving the leading coefficient 2 in place. First divide the whole equation by 2, then use the new coefficient 4.

LESSON 35 | REWRITE A FUNCTION WITHOUT CHANGING IT

The same idea reveals the vertex

For a function rule, keep the expression equivalent by **adding and subtracting** the same number inside it. This adds zero overall.

Rewrite $y = x^2 + 6x + 2$ in vertex form

$y = x^2 + 6x + 9 - 9 + 2$

$y = (x + 3)^2 - 7$

Add and subtract 9, the square of half of 6.

The square reveals the vertex $(-3, -7)$.

Your task	What preserves the value or equality?
Solve $x^2 + 6x = 7$	Add 9 to BOTH sides: $x^2 + 6x + 9 = 16$.
Rewrite $x^2 + 6x + 2$	Add 9 and subtract 9 within the same expression.

A common error to catch

$x^2 + 6x = 7$ does **not** become $(x + 3)^2 = 7$.

Expanding that square adds 9 to the left side. The right side must also gain 9, giving 16.

Completing the square always creates an equivalent equation when you use balanced operations. The square-root step then tells you whether there are two, one or no real solutions.

LESSON 35 | SHORT CHECKS

Your turn

1. Fill the blank and factor: $x^2 + 10x + \underline{\hspace{2cm}}$.

2. Solve $x^2 + 4x = 12$ by completing the square.

3. Solve $x^2 - 6x = 7$ by completing the square.

4. Solve $2x^2 + 4x - 4 = 0$. Divide first, then complete the square.

5. A student changes $x^2 + 8x = 5$ into $(x + 4)^2 = 5$. Correct the equation and finish solving.

LESSON 35 | WORKED ANSWERS

Check the added term on both sides

1. Half of 10 is 5, and $5^2 = 25$. Thus $x^2 + 10x + 25 = (x + 5)^2$.

2. Add 4 to both sides: $x^2 + 4x + 4 = 16$.

$(x + 2)^2 = 16$, so $x + 2 = \pm 4$ and $x = 2$ or -6 .

Checks: $4 + 8 = 12$ and $36 - 24 = 12$.

3. Add 9: $x^2 - 6x + 9 = 16$.

$(x - 3)^2 = 16$, so $x = 7$ or -1 .

Checks: $49 - 42 = 7$ and $1 + 6 = 7$.

4. Divide by 2: $x^2 + 2x = 2$. Add 1: $(x + 1)^2 = 3$.

$x = -1 \pm \sqrt{3}$. Check the original as $2[(x + 1)^2 - 3] = 2(3 - 3) = 0$.

5. Add 16 to **both** sides: $(x + 4)^2 = 21$.

Then $x = -4 \pm \sqrt{21}$. Check: $x^2 + 8x = (x + 4)^2 - 16 = 21 - 16 = 5$.

I can...

- ☐ Make the leading coefficient 1 before using the half-coefficient rule.
- ☐ Add the same square to both sides of an equation.
- ☐ Connect a completed square to vertex form.

Next class: the quadratic formula and real-root decisions. Curriculum focus: A.8A.