

LESSON 11 | MEET THE IDEA

Linear domains, ranges and direct variation

Tuesday, November 17, 2026 . 6:00–7:00 PM Central

By the end, I can...

Choose sensible inputs and outputs, distinguish continuous values from counts, and build direct variation rules.

Word	Meaning	Question to ask
Domain	The allowed inputs.	Which x-values can I use?
Range	The outputs produced by those inputs.	Which y-values actually occur?
Continuous	All values across an interval, including fractions.	Is this a measurement?
Discrete	Separate allowed values.	Is this a whole-number count?

Example 1 | $y = 2x + 1$ with no stated restriction

Domain: all real numbers

Any real input can be multiplied by 2 and increased by 1.

Range: all real numbers

Every real output is reached: solve $x = (y - 1)/2$.

Remember the horizontal exception

For $y = 5$ with no input restriction, the domain is all real numbers, but the range is **only 5**. A constant line does not reach every output.

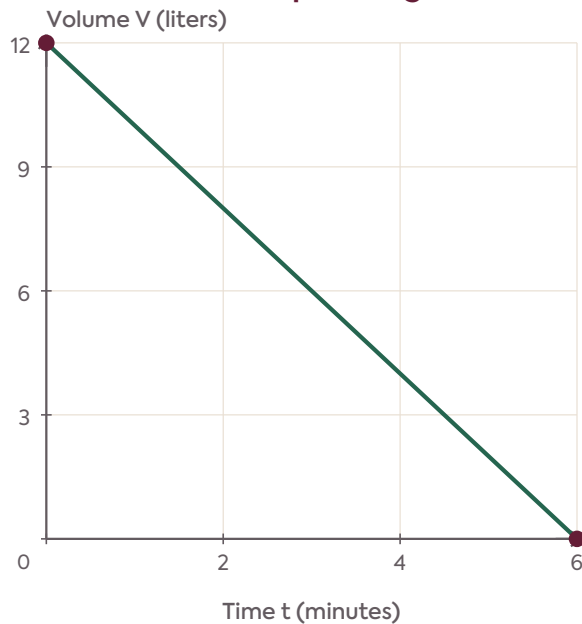
LESSON 11 | CONTINUOUS AND DISCRETE MODELS

A situation can restrict a line

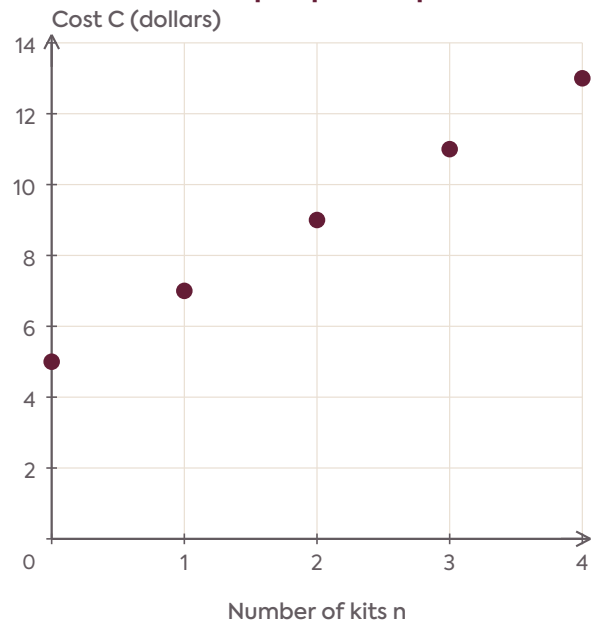
Tank: $V = 12 - 2t$ from the start until empty. Time is continuous: $0 \leq t \leq 6$. The volume range is $0 \leq V \leq 12$.

Club: $C = 5 + 2n$ for buying up to four project kits. n must be a whole number: $n = 0, 1, 2, 3, 4$. The costs are **5, 7, 9, 11, 13 dollars**.

Continuous: keep the segment



Discrete: keep separate points



Bounds alone are not enough

For the club, $0 \leq n \leq 4$ must be accompanied by “ **n is a whole number**”. Similarly, $5 \leq C \leq 13$ alone would wrongly suggest that \$6 is possible. List the outputs or describe their \$2 steps.

The tank graph includes fractional minutes such as 2.5. The kit graph excludes fractional counts such as 2.5. That is why one graph is joined and the other is not.

LESSON 11 | WRITE $y = kx$

Direct variation: one constant rate

In **direct variation**, $y = kx$. For each nonzero input, y/x has the same value k . The unrestricted graph is a straight line through the origin. Real contexts may use only part of that line.

Example 2 | Fabric costs \$14 for 4 meters, with no fixed fee

$$k = 14/4 = 3.5$$

Divide cost by length: \$3.50 per meter.

$$C = 3.5l$$

l is length in meters; C is cost in dollars.

$$C = 3.5(6) = 21$$

Six meters cost \$21 under this constant-price model.

For continuous purchases, $l \geq 0$. The model gives $C = 0$ at $l = 0$.

Example 3 | y varies directly with x ; $y = 18$ when $x = 6$

$$k = 18/6 = 3$$

Find the constant before finding a new output.

$$y = 3x$$

Write the complete relationship.

$$27 = 3x, \text{ so } x = 9$$

When $y = 27$, solve for the input.

A constant slope is not the whole test

$C = 3.5l + 5$ has constant slope, but the \$5 fixed fee means it is **not** direct variation. Its unrestricted graph misses the origin.

LESSON 11 | SHORT CHECKS

Your turn

1. State the domain and range of $y = -3x + 7$ when there are no restrictions.

2. For $y = 2x + 1$, the domain is $x = 0, 1, 2, 3$ only. List the range. Should you join these points on a graph?

3. A tank model is $V = 20 - 4t$, for $0 \leq t \leq 5$. Time t is continuous. Give the domain and range, with units.

4. y varies directly with x . When $x = 5$, $y = 20$. Find the rule and find y when $x = 8$.

5. A club charges \$5 to join plus \$2 per visit. Is the cost directly proportional to the number of visits? Explain.

6. For $y = 4$ with any real input, what is the range?

Say both parts

State the bounds **and** the kind of values allowed. Use a list when an inequality would include outputs that the context cannot produce.

LESSON 11 | WORKED ANSWERS AND RECAP

Check your thinking

- 1. Domain: all real numbers. Range: all real numbers.** The nonzero slope means every real output can be reached by some real input.
- 2.** The range is **1, 3, 5, 7**. Do **not** join the points: no inputs between the listed values are allowed.
- 3.** Domain: **$0 \leq t \leq 5$ minutes**, including fractional times. Range: **$0 \leq V \leq 20$ liters**. At $t = 0$, $V = 20$; at $t = 5$, $V = 0$.
- 4.** $k = 20/5 = 4$, so **$y = 4x$** . When $x = 8$, $y = 4(8) = 32$.
- 5. No.** The rule is $C = 2n + 5$, so cost is \$5 when the visit count is 0. A direct variation rule would have the form $C = kn$, with no added fixed fee.
- 6.** The range contains **only 4**. Changing the input never changes the output.

I can...

- ☐ State domain and range accurately.
- ☐ Distinguish measurements from discrete counts.
- ☐ Find k and solve a direct variation problem.

Next class: transform the parent line $f(x) = x$. Curriculum focus: A.2A, A.2D.