

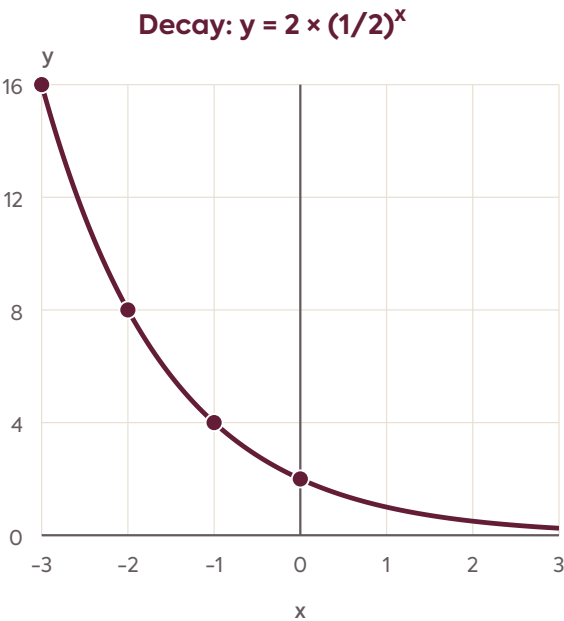
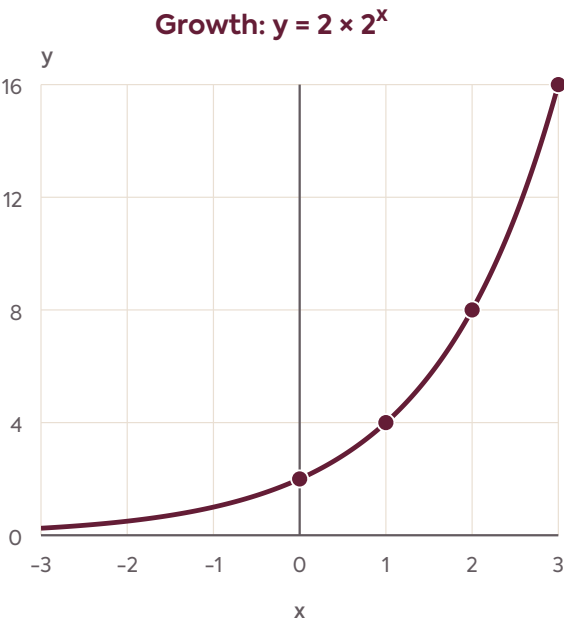
LESSON 42 | MEET THE IDEA

Exponential graphs, domains, ranges and asymptotes

Thursday, April 1, 2027 . 6:00–7:00 PM Central

By the end, I can...
Graph exponential functions, identify intercepts and asymptotes, and describe domains and ranges that match the context.

Begin with a table, then draw a smooth curve if the input is continuous. These two unrestricted functions have positive starting values and the same y-intercept (0, 2).



x	-2	-1	0	1	2
2×2^x	1/2	1	2	4	8
$2 \times (1/2)^x$	8	4	2	1	1/2

Both curves continue beyond the windows. For a positive starting value, $b > 1$ gives an increasing curve; $0 < b < 1$ gives a decreasing curve. The marked dots help draw the curve; they do not limit its domain.

LESSON 42 | DOMAIN, RANGE AND ASYMPTOTE

Approach zero without reaching it

For an unrestricted $f(x) = ab^x$
Assume **$a \neq 0$, $b > 0$ and $b \neq 1$** .
Domain: all real x .
y-intercept: **$(0, a)$** , because $b^0 = 1$.
Horizontal asymptote: **$y = 0$** .
No x-intercept: b^x is positive, so ab^x cannot equal zero.

A **horizontal asymptote** is a horizontal line the curve approaches in an end direction. For $f(x) = 2(1/2)^x$, the outputs approach 0 as x becomes very large, but they remain positive.

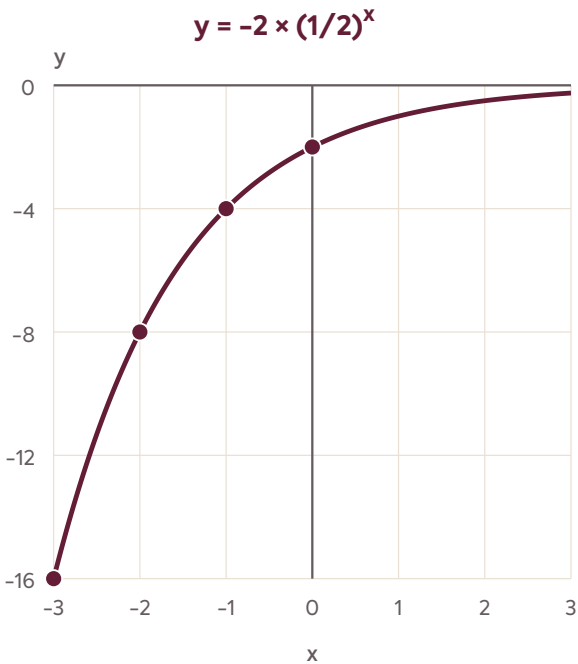
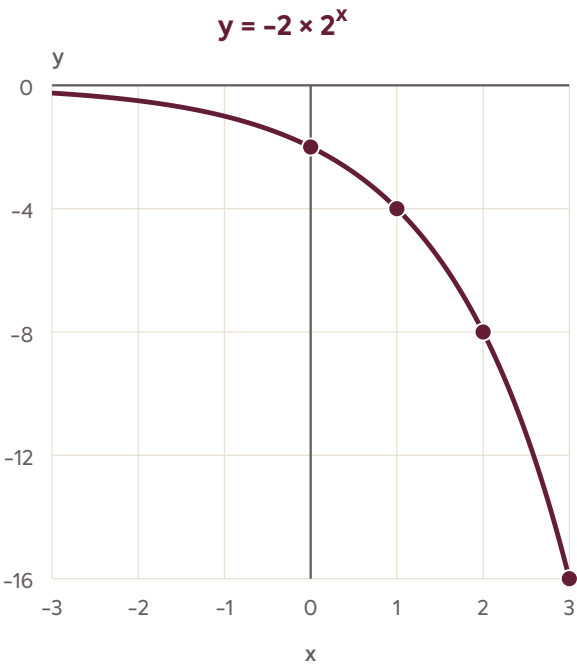
x	4	6	8	10
$2(1/2)^x$	0.125	0.03125	0.0078125	0.001953125

The sign of a determines the unrestricted range
If **$a > 0$** , the range is **$y > 0$** , or $(0, \infty)$.
If **$a < 0$** , the range is **$y < 0$** , or $(-\infty, 0)$.
Zero is excluded in both cases. Parentheses mean the endpoint is not included.

A calculator may round a tiny output to 0 or make the curve appear to touch the axis. That display does not make the exact exponential value zero. These rules are for the unshifted form ab^x .

LESSON 42 | KEEP THE BASE POSITIVE

A negative a reflects the curve



Multiplying the whole function by -1 reflects it across the x -axis. The **base is still positive**. Both graphs have y -intercept $(0, -2)$, domain all real numbers, range $y < 0$ and asymptote $y = 0$.

Function	As x increases	What approaches zero?
-2×2^x	Decreases: $-2, -4, -8, \dots$	Outputs approach 0 from below as x heads toward $-\infty$.
$-2 \times (1/2)^x$	Increases: $-2, -1, -1/2, \dots$	Outputs approach 0 from below as x heads toward $+\infty$.

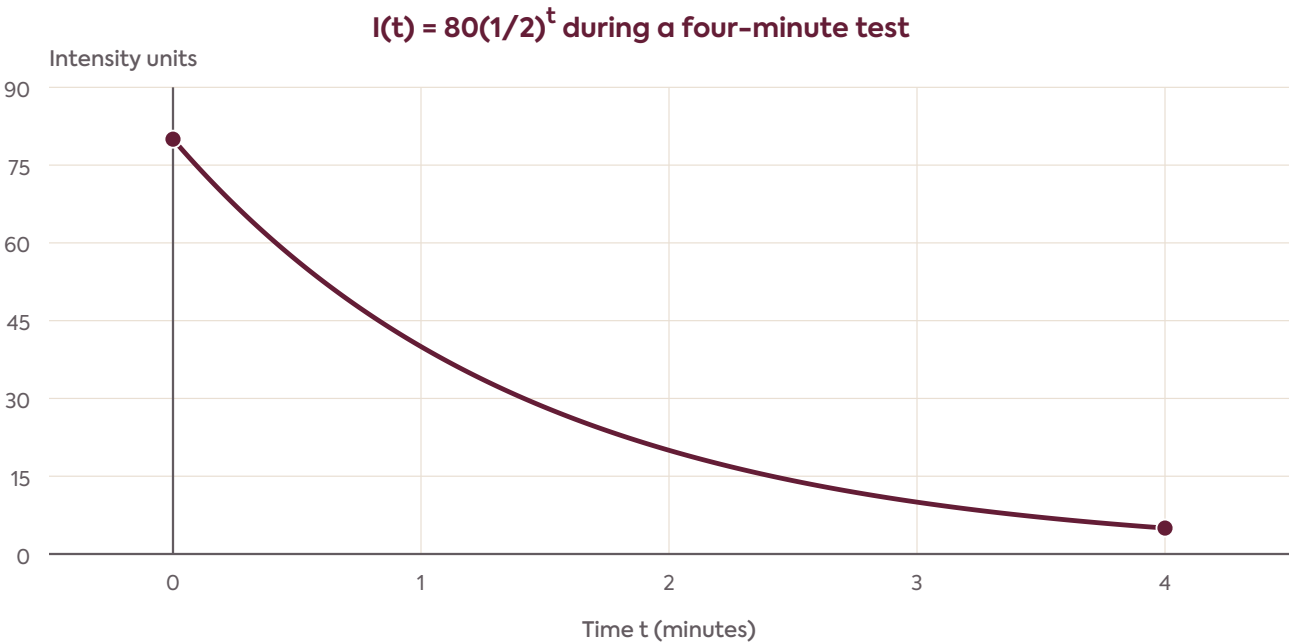
Coefficient sign is not base sign

-2×2^x has positive base 2 and is real for every real x .

$2 \times (-2)^x$ has negative base -2 and is not a standard real exponential function on all real inputs.

LESSON 42 | RESTRICT THE GRAPH AND THE RANGE TOGETHER

The context can shrink the domain



During the test, $0 \leq t \leq 4$. Intensity decreases continuously from 80 to 5, so the model range is $5 \leq I \leq 80$. Filled endpoint dots show that both endpoints are included.

Allowed inputs for the same formula	Corresponding outputs
Every real t	All positive values: $I > 0$
Continuous time $t \geq 0$	$0 < I \leq 80$
Only readings $t = 0, 1, 2, 3, 4$	Exactly $\{80, 40, 20, 10, 5\}$

Do not give the unrestricted range for a restricted story

The four-minute segment never gets below 5. The asymptote $y = 0$ describes the extended function, not a value reached during the test. Discrete readings include only listed outputs, not every value between them.

LESSON 42 | SHORT CHECKS

Your turn

1. For $f(x) = 3 \times 2^x$, find $f(-1)$, $f(0)$, $f(1)$ and $f(2)$. Is it increasing or decreasing?

2. For unrestricted $g(x) = 5(1/3)^x$, state domain, range, y-intercept, horizontal asymptote and any x-intercept.

3. For $h(x) = -4 \times 2^x$, state its range and y-intercept. Does h increase or decrease as x increases?

4. The model $I(t) = 64(1/2)^t$ applies for continuous time $0 \leq t \leq 3$. State its domain and range.

5. For the same model, suppose only $t = 0, 1, 2, 3$ readings are allowed. List the domain and range. Should the dots be connected?

6. A calculator displays 0 for $7(1/2)^{100}$. Is the exact value zero? Explain using the formula.

LESSON 42 | WORKED ANSWERS

Check the allowed inputs first

1. The outputs are **1.5, 3, 6 and 12**. The function is **increasing**: $a = 3 > 0$ and $b = 2 > 1$.
2. Domain: **all real numbers**. Range: **$y > 0$** . y-intercept: **(0, 5)**. Horizontal asymptote: **$y = 0$** . There is **no x-intercept** because a positive exponential output never equals zero.
3. Range: **$y < 0$** . y-intercept: **(0, -4)**. The function **decreases**; for example, $h(0) = -4$, $h(1) = -8$ and $h(2) = -16$. "Increasing base" does not override a negative coefficient.
4. Domain: **$0 \leq t \leq 3$** . $l(0) = 64$ and $l(3) = 8$. The function decreases continuously, so its range is **$8 \leq l \leq 64$** .
5. Domain: **{0, 1, 2, 3}**. Range: **{64, 32, 16, 8}**. Use four separate dots; connecting them would include times and outputs outside the discrete reading set.
6. **No.** $7(1/2)^{100} = 7/2^{100}$ is positive. The calculator display is rounded; the exact function has not reached its asymptote.

Two boundary cases

If $a = 0$, $f(x) = 0$ is the zero function. If $b = 1$, $f(x) = a$ is constant. These are outside the nonconstant exponential cases studied here.

I can...

- ☐ Use coefficient sign to decide an unrestricted range.
- ☐ Distinguish a plotted curve from discrete readings.
- ☐ Restrict both domain and range to the context.

Next class: fit a model to data and review Chapter 7. Curriculum focus: A.9A and A.9D; review A.9B-C.