

LESSON 27 | MEET THE IDEA

Common factors and monic quadratic trinomials

Tuesday, February 2, 2027 . 6:00–7:00 PM Central

By the end, I can...

Rewrite a sum as a product, take out a greatest common factor, and factor a monic quadratic using a product-and-sum check.

Factoring reverses multiplication. Expanding turns a product into a sum. Factoring turns that sum back into a product with the same value.

Read the distributive property in both directions

$$3x(x + 4) = 3x^2 + 12x$$

Expand: multiply $3x$ by each term in the parentheses.

$$3x^2 + 12x = 3x(x + 4)$$

Factor: both terms contain $3x$. Divide each term by that common factor.

Find the greatest common factor (GCF)

Use the greatest common numerical factor and the lowest exponent of every variable shared by **all** terms.

Factor $12x^3 + 18x^2$

$$\text{GCF} = 6x^2$$

12 and 18 share 6; x^3 and x^2 share two factors of x .

$$6x^2(2x + 3)$$

$$12x^3 \div 6x^2 = 2x; 18x^2 \div 6x^2 = 3.$$

Check by expansion: $6x^2(2x) + 6x^2(3) = 12x^3 + 18x^2$. The factored identity is valid for every real x , including 0.

LESSON 27 | MONIC QUADRATIC TRINOMIALS

Two numbers do two jobs

A **quadratic trinomial** has three terms and degree 2. **Monic** means the coefficient of x^2 is 1.

Why the product-and-sum method works

$(x + m)(x + n)$	Multiply every term by every term.
$x^2 + nx + mx + mn$	The middle two terms are like terms.
$x^2 + (m + n)x + mn$	The sum $m + n$ is the x-coefficient; the product mn is the constant.

For $x^2 + bx + c$

Find numbers m and n with $mn = c$ and $m + n = b$.
Then $x^2 + bx + c = (x + m)(x + n)$.

Factor pair for 12	Sum	Does it give 7?
1 and 12	13	No
2 and 6	8	No
3 and 4	7	Yes

So $x^2 + 7x + 12 = (x + 3)(x + 4)$.
Check: $x^2 + 4x + 3x + 12 = x^2 + 7x + 12$.

LESSON 27 | NEGATIVE TERMS AND GCF FIRST

Let the signs guide your search

Constant c	What to look for
Positive	Same signs. A positive sum needs two positives; a negative sum needs two negatives.
Negative	Opposite signs. Their sum must still equal the x-coefficient.

Factor $x^2 - x - 12$

$m \times n = -12; m + n = -1$
 $m = -4; n = 3$
 $(x - 4)(x + 3)$

The numbers must have opposite signs.

$(-4)(3) = -12$ and $-4 + 3 = -1$.

Check: $x^2 + 3x - 4x - 12 = x^2 - x - 12$.

Factor $2x^2 - 10x + 12$ completely

$2(x^2 - 5x + 6)$
 $2(x - 2)(x - 3)$

Take out the GCF, 2, before using the monic method.

The pair -2 and -3 has product 6 and sum -5.

Check: $2(x^2 - 5x + 6) = 2x^2 - 10x + 12$. Keep the factor 2 in the final answer.

Factor an expression; do not invent an equation

An expression such as $x^2 - x - 12$ has no equals sign. The task is to rewrite it, not to find x. Solving a quadratic equation comes in Chapter 6.

LESSON 27 | SHORT CHECKS

Your turn

Factor each expression completely over the integers. Check by expanding. For Question 6, explain the error in words.

1. $8x^2 + 12x$

2. $x^2 + 9x + 20$

3. $x^2 - 7x + 10$

4. $x^2 + 2x - 15$

5. $3x^2 - 3x - 18$

6. Sam says $x^2 + 6x + 8 = (x + 1)(x + 8)$ because $1 \times 8 = 8$. What did Sam miss?

LESSON 27 | WORKED ANSWERS

Check both the product and the sum

1. $4x(2x + 3)$. The GCF is $4x$; expanding gives $8x^2 + 12x$.
2. $(x + 4)(x + 5)$. $4 \times 5 = 20$ and $4 + 5 = 9$. The cross-products give $5x + 4x = 9x$.
3. $(x - 5)(x - 2)$. $(-5)(-2) = 10$ and $-5 + (-2) = -7$. Expanding gives $x^2 - 7x + 10$.
4. $(x + 5)(x - 3)$. $5(-3) = -15$ and $5 - 3 = 2$. The two signs must be different.
5. $3(x^2 - x - 6) = 3(x - 3)(x + 2)$. Keep the GCF 3. Expansion gives $3x^2 - 3x - 18$.
6. Sam did not check the **sum**: $1 + 8 = 9$, not 6. Use 2 and 4, so the correct factors are $(x + 2)(x + 4)$.

If no integer pair works

Do not force an answer. For $x^2 + x - 1$, no integer pair has product -1 and sum 1 . This does **not** prove that no real factors exist. Later lessons explain non-integer real factors.

I can...

- ☐ Take out a GCF before factoring further.
- ☐ Check the product, sum and signs of a factor pair.
- ☐ Verify a factorization by expanding.

Next class: quadratics whose leading coefficient is not 1. Curriculum focus: A.10D, A.10E.