

LESSON 28 | MEET THE IDEA

Nonmonic and perfect-square trinomials

Thursday, February 4, 2027 . 6:00–7:00 PM Central

By the end, I can...

Factor a nonmonic quadratic using one reliable grouping method, and recognize a perfect-square trinomial by checking all three terms.

In a **nonmonic** quadratic $ax^2 + bx + c$, the leading coefficient a is not 1. After taking out any GCF, we will **split the middle term and group**.

One method: multiply, split, group, check

1. Calculate ac .
2. Find two integers with product ac and sum b .
3. Replace bx with those two x -terms.
4. Factor the two groups, then their common binomial.
5. Expand to check.

Factor $6x^2 + 11x + 3$

$$ac = 6 \times 3 = 18$$

$$6x^2 + 9x + 2x + 3$$

$$3x(2x + 3) + 1(2x + 3)$$

$$(3x + 1)(2x + 3)$$

Find a pair with product 18 and sum 11: 9 and 2.

Splitting $11x$ into $9x + 2x$ keeps the expression equivalent.

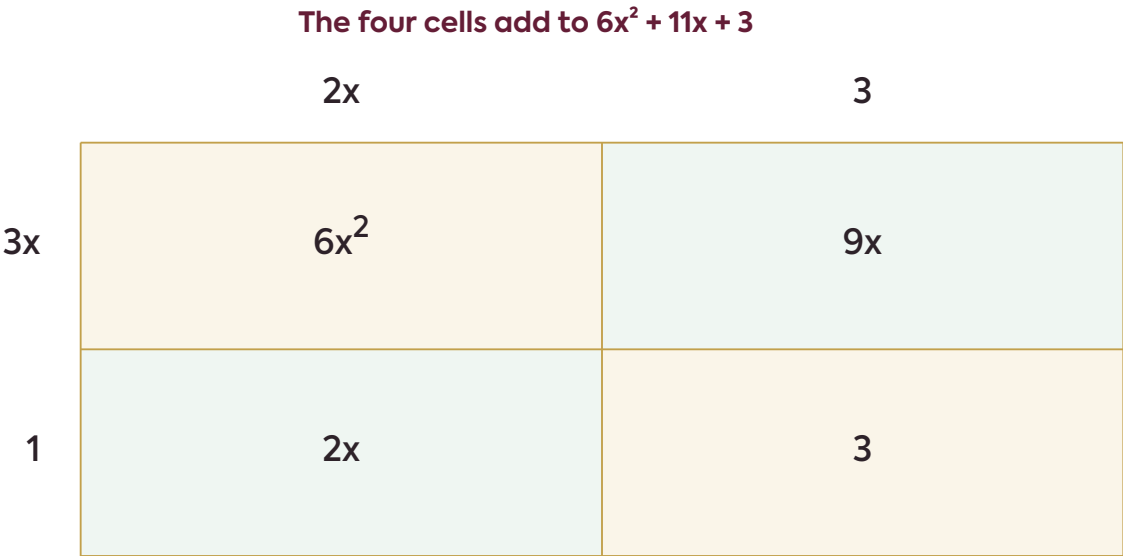
Factor each group. The binomials match.

Factor out the shared binomial, $2x + 3$.

Notice: 9 and 2 split the middle term. They are not automatically the constants in the final factors.

LESSON 28 | THE SAME EXAMPLE, ORGANIZED VISUALLY

See the groups in a grid



This is an algebra grid, not a scale drawing.

Each cell is its row label multiplied by its column label. Add the row labels to get $3x + 1$; add the column labels to get $2x + 3$. The grid records the same grouping method.

Multiply back to verify

$(3x + 1)(2x + 3)$

$6x^2 + 9x + 2x + 3$

$6x^2 + 11x + 3$

Every term meets every term.

The middle two products add to $11x$.

The original trinomial is recovered.

When the groups do not match

Recheck the pair, signs and GCFs. You may reorder the two split terms. Do not drop a term or change a sign just to make matching parentheses.

LESSON 28 | KEEP EVERY SIGN

Use grouping with negative terms

Factor $6x^2 - x - 2$

$$ac = -12; b = -1$$

$$6x^2 - 4x + 3x - 2$$

$$2x(3x - 2) + 1(3x - 2)$$

$$(2x + 1)(3x - 2)$$

Use -4 and 3: their product is -12 and their sum is -1.

Split the middle term into $-4x + 3x$.

Factor each pair.

Check: $6x^2 - 4x + 3x - 2 = 6x^2 - x - 2$.

Factor $2x^2 - 7x + 3$

$$ac = 6; b = -7$$

$$2x^2 - 6x - x + 3$$

$$2x(x - 3) - 1(x - 3)$$

$$(2x - 1)(x - 3)$$

Use -6 and -1: same signs, negative sum.

Split $-7x$ into $-6x - x$.

Take out -1 from the second group: $-x + 3 = -1(x - 3)$.

Check: $2x^2 - 6x - x + 3 = 2x^2 - 7x + 3$.

LESSON 28 | CHECK THE MIDDLE TERM

A trinomial that is a square

A **perfect-square trinomial** is the product of a binomial with itself. The first and last terms being squares is not enough: the middle term must also match.

The two patterns

$$u^2 + 2uv + v^2 = (u + v)^2$$

$$u^2 - 2uv + v^2 = (u - v)^2$$

Factor $9x^2 - 24x + 16$

$u = 3x; v = 4$

$-2(3x)(4) = -24x$

$(3x - 4)^2$

$(3x)^2 = 9x^2$ and $4^2 = 16$.

The middle term matches the minus pattern.

Check by expanding $(3x - 4)(3x - 4)$.

Trinomial	Decision
$x^2 + 10x + 25$	$(x + 5)^2$, since $2(x)(5) = 10x$.
$4x^2 + 12x + 9$	$(2x + 3)^2$, since $2(2x)(3) = 12x$.
$4x^2 + 10x + 9$	Not $(2x + 3)^2$: that expansion has $12x$, not $10x$.

Always look for a GCF first: $2x^2 + 12x + 18 = 2(x^2 + 6x + 9) = 2(x + 3)^2$.

LESSON 28 | SHORT CHECKS

Your turn

Use grouping for Questions 1–3. Factor completely in Questions 4–5. Explain your decision in Question 6.

1. $2x^2 + 7x + 3$

2. $3x^2 - 5x - 2$

3. $6x^2 - 7x + 2$

4. $4x^2 - 20x + 25$

5. $2x^2 + 12x + 18$

6. Is $9x^2 + 12x + 4$ equal to $(3x + 2)^2$? What about $9x^2 + 10x + 4$?

LESSON 28 | WORKED ANSWERS

Check the middle term every time

1. $ac = 6$; split $7x$ as $6x + x$.

$$2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3).$$

2. $ac = -6$; split $-5x$ as $-6x + x$.

$$3x^2 - 6x + x - 2 = 3x(x - 2) + 1(x - 2) = (3x + 1)(x - 2).$$

3. $ac = 12$; split $-7x$ as $-4x - 3x$.

$$6x^2 - 4x - 3x + 2 = 2x(3x - 2) - 1(3x - 2) = (2x - 1)(3x - 2).$$

4. $(2x - 5)^2$. The middle term is $-2(2x)(5) = -20x$; the last term is 25.

5. Take out 2 first: $2(x^2 + 6x + 9) = 2(x + 3)^2$.

6. **Yes** for $9x^2 + 12x + 4$ because $2(3x)(2) = 12x$. **No** for $9x^2 + 10x + 4$: its middle term does not match the claimed square.

I can...

- ☐ Use the ac pair to split the middle term.
- ☐ Factor a negative group without losing a sign.
- ☐ Recognize a square by checking its middle term.

Next class: differences of squares and selecting a factoring method. Curriculum focus: A.10E.