



CHAPTER 03

Algebra I

Linear inequalities and systems

Classes 15–20 . December 8, 2026–January 7, 2027
Tuesdays and Thursdays . 6:00–7:00 PM Central

MAKE BOTH CONDITIONS TRUE

$$y = x + 1$$

A relationship with many points.

$$y = -x + 5$$

A second relationship on the same axes.

$$(2, 3)$$

One point that satisfies both equations.

Clear explanations . Worked examples . Accurate graphs
Short practice checks . Worked answers . Chapter revision

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Your route through Chapter 3

Open your next lesson or revisit a topic. Each lesson includes explanations, worked examples and short practice checks with answers.

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Chapter quick reference: page 36 . Study guidance and sources: page 37

Use the lesson pages during each 60-minute class and for revision afterward. Complete the digital practice your teacher assigns separately.

LESSON 15 | MEET THE IDEA

Two-variable linear inequalities

Tuesday, December 8, 2026 . 6:00–7:00 PM Central

By the end, I can...

Write a linear inequality in two variables, graph its solution region and explain whether a point belongs.

An equation such as $x + y = 4$ describes a line. An inequality such as $x + y \leq 4$ also allows points on one side of that line. Its solutions are **ordered pairs** (x, y) .

Point	Substitute in $x + y \leq 4$	Solution?
(1, 2)	$1 + 2 = 3$, and $3 \leq 4$.	Yes
(1, 3)	$1 + 3 = 4$, and $4 \leq 4$.	Yes: on the boundary
(3, 2)	$3 + 2 = 5$, and $5 \leq 4$ is false.	No

Four steps for the graph

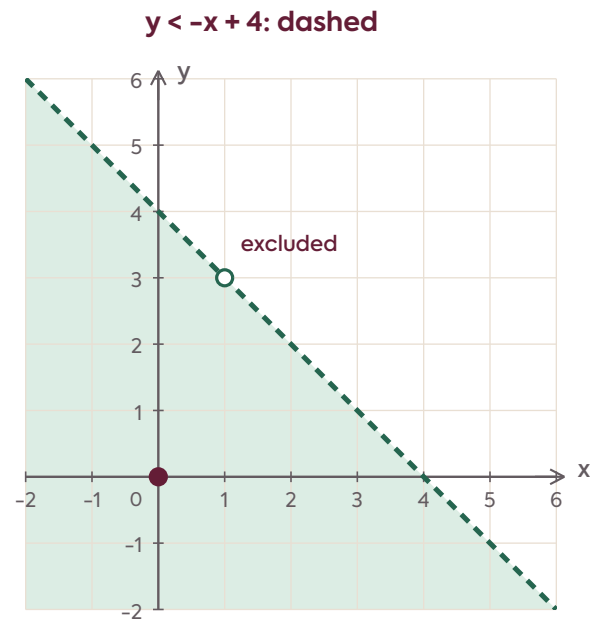
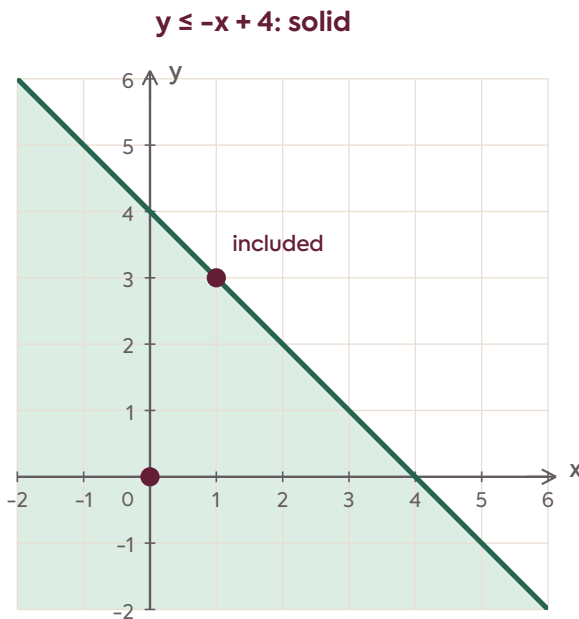
1. Replace the inequality sign with $=$ to draw the boundary.
2. Use solid for $\leq$ or $\geq$; dashed for $<$ or $>$.
3. Test a point that is not on the boundary.
4. Shade the side that makes the original inequality true.

A straight boundary divides the plane into two half-planes. Points on the same side give the same true/false result, which is why one test point can choose the side.

LESSON 15 | SEE THE DIFFERENCE

Does the boundary count?

Both graphs shade below $y = -x + 4$. Only the left graph includes the boundary line. The shaded regions continue beyond the grids shown.



Test $(0, 0)$: $0 < 4$ and $0 \leq 4$ are both true, so shade the side containing the origin. A point on the boundary cannot choose the side.

Be careful when you divide by a negative number

$$x - 2y < 4$$

Subtract x from both sides.

$$-2y < 4 - x$$

Divide both sides by -2 and reverse the sign.

$$y > (1/2)x - 2$$

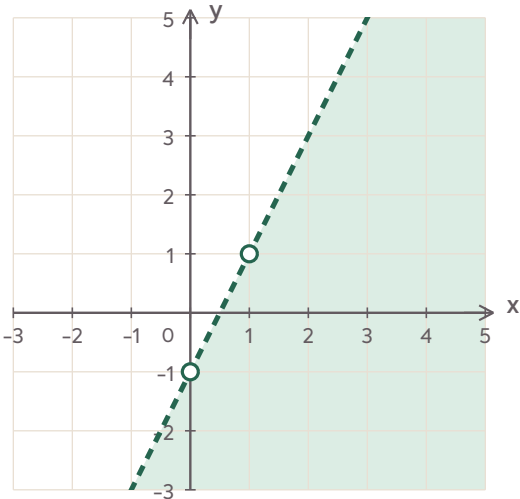
Use a dashed boundary and shade above it.

LESSON 15 | GRAPHS AND TABLES

Write the rule from evidence

Read this graph: the dashed boundary passes through (0, -1) and (1, 1). The shaded side lies below it. What inequality does it show?

Read the boundary and shaded side



Slope = $(1 - (-1)) / (1 - 0) = 2$; y-intercept = -1. Dashed means equality is excluded. Shading below gives $y < 2x - 1$.

For boundary $x + y = 4$	Inside shaded region?
(0, 0)	Yes
(1, 3)	Yes, including the boundary
(3, 2)	No

Choose from $x + y < 4$, $x + y \leq 4$, $x + y > 4$, $x + y \geq 4$. The table selects $x + y \leq 4$. Without the supplied boundary and choices, a few points would not determine a unique inequality.

LESSON 15 | A REAL-WORLD MODEL

Turn a budget into a region

A notebook costs \$3 and a pen costs \$2. You can spend **at most \$12**. Let x be notebooks and y be pens. The budget rule is $3x + 2y \leq 12$, with x and y nonnegative whole numbers.

Every dot is an affordable whole-number choice



The solid boundary shows spending exactly \$12, through (0, 6) and (4, 0). Shading below shows spending no more than \$12. **Only the dots** represent allowed purchases here; you cannot buy a fraction of a notebook.

Choice	Cost	Allowed?
(2, 3)	$\$6 + \$6 = \$12$	Yes
(3, 2)	$\$9 + \$4 = \$13$	No

Translate carefully
At most / no more than: $\leq$. At least / no less than: $\geq$.
Fewer than / less than: $<$. More than: $>$. Define your variables and units before choosing a symbol.

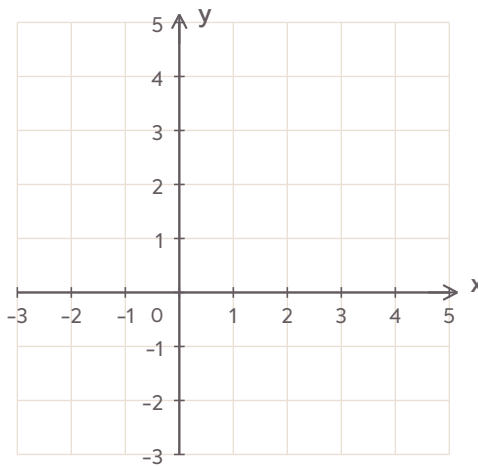
LESSON 15 | SHORT CHECKS

Your turn

Try these before reading the worked answers. Your main homework is in digital practice.

1. Graph $y > x - 1$ on the grid. Mark the boundary correctly and test $(0, 0)$.

Your solution region



2. Which points satisfy $x + 2y \leq 6$: $(2, 2)$, $(4, 2)$, $(0, 3)$? Show the substitutions.

3. Rearrange $x - 2y \leq 4$ to make y the subject. Is its region above or below the boundary?

4. Small kits cost \$4 and large kits cost \$7. Write the rule for spending at most \$28. Include sensible restrictions on the numbers of kits.

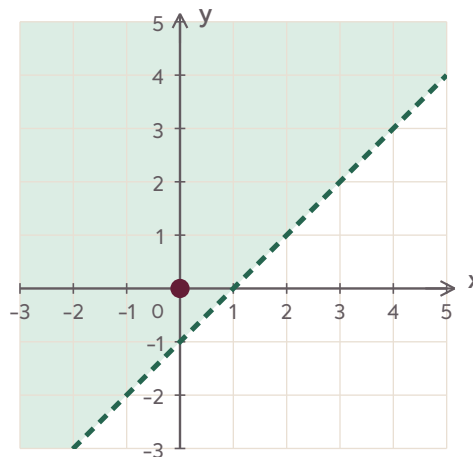
5. A solid boundary passes through $(0, 2)$ and $(2, 0)$; the shaded side includes $(0, 0)$. Write the inequality.

LESSON 15 | WORKED ANSWERS

Check the side, sign and meaning

1. Draw $y = x - 1$ **dashed** and shade above it. At $(0, 0)$, $0 > -1$ is true. The origin belongs; boundary points do not.

Answer 1: $y > x - 1$



2. **(2, 2): yes**, because $2 + 2(2) = 6 \leq 6$. **(4, 2): no**, because $8 > 6$. **(0, 3): yes**, because $6 \leq 6$. The two solutions lie on the included boundary.

3. $-2y \leq 4 - x$ becomes $y \geq (1/2)x - 2$. Reverse the sign when dividing by -2 ; use a solid boundary and shade above.

4. Let s and l count small and large kits. $4s + 7l \leq 28$, where s and l are nonnegative whole numbers.

5. The boundary is $y = -x + 2$. Solid and below gives $y \leq -x + 2$, or $x + y \leq 2$.

I can...

- ☐ Choose a boundary and shade the correct side.
- ☐ Test a point in the original inequality.

Next class: two equations that must be true together. Curriculum focus: A.2H, A.3D.

LESSON 16 | MEET THE IDEA

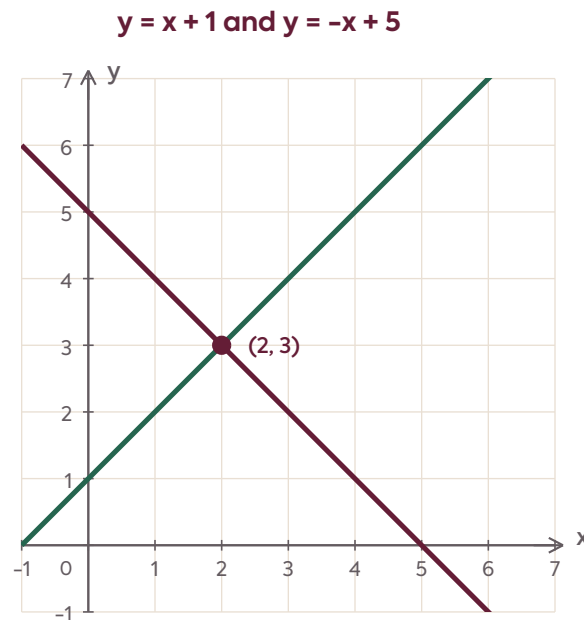
Writing and graphing systems of equations

Thursday, December 10, 2026 . 6:00–7:00 PM Central

By the end, I can...

Write two equations for the same situation and use their graphs to find and interpret a shared solution.

A **system** is a set of equations that must be true at the same time. For two lines, a shared solution is a point on **both** lines.



Rising line: $y = x + 1$. Falling line: $y = -x + 5$.

The intersection is the answer

The graph suggests **(2, 3)**. Check both equations:

$3 = 2 + 1$, and $3 = -2 + 5$. Both are true, so the coordinates are exact.

Report an ordered pair, not just $x = 2$. The input and output describe the same point.

LESSON 16 | TABLES, GRAPHS AND WORDS

Build the two rules

These tables describe two **linear** relationships. Use each row of outputs to find its own slope and intercept.

x	0	1	2
First output	1	3	5
Second output	7	6	5

From the table to a system

$$y = 2x + 1$$

$$y = -x + 7$$

Shared point: (2, 5)

First output: add 2 per input unit; start at 1.

Second output: subtract 1 per input unit; start at 7.

Both tables give $y = 5$ when $x = 2$. Check both rules.

From a graph

On the opening graph, one line passes through (0, 1) and (2, 3), so its slope is 1. The other passes through (0, 5) and (2, 3), so its slope is -1. Write **one equation for each line**.

From words

A club has 12 tickets to sell: adult tickets a and student tickets s . Adult tickets cost \$8 and student tickets cost \$5. Total sales are \$78.

$a + s = 12$ counts tickets.

$8a + 5s = 78$ counts dollars.

Keep the meaning consistent

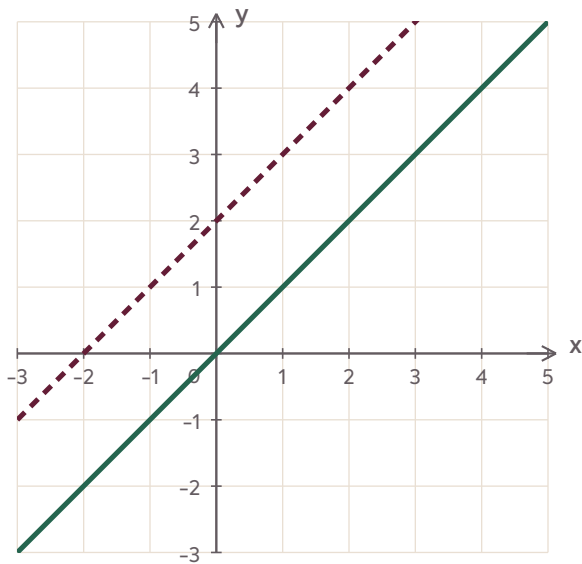
The same variable must mean the same quantity in both equations. Do not add a ticket count to a dollar amount.

LESSON 16 | WHAT THE LINES SHOW

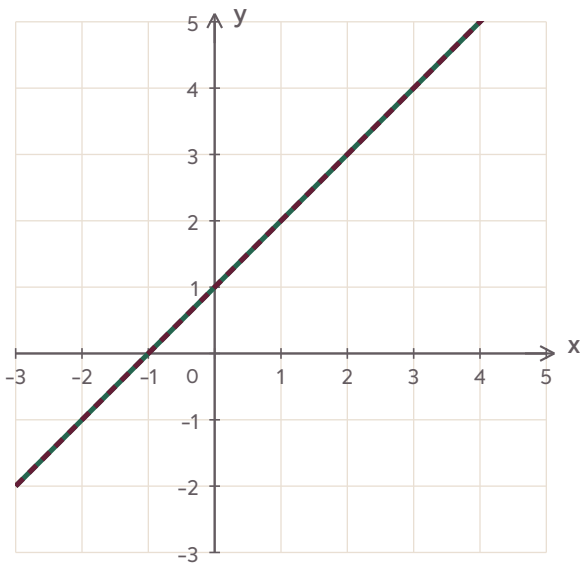
One, none or infinitely many?

Two straight lines can cross once, stay separate forever, or lie exactly on top of each other. The picture tells you how many shared solutions they have.

Distinct parallel lines: none



Same line: infinitely many



Case	Equations	Shared solutions
Cross once	$y = x + 1$; $y = -x + 5$	One: $(2, 3)$.
Same slope, different intercepts	$y = x$; $y = x + 2$	None: there is no intersection.
Equivalent equations	$y = x + 1$; $2y = 2x + 2$	Every point on their shared line.

Here, a dashed stroke helps distinguish one **equation** from the other; both equation graphs include every point on their lines. Only an **inequality** uses a dashed boundary to exclude equality.

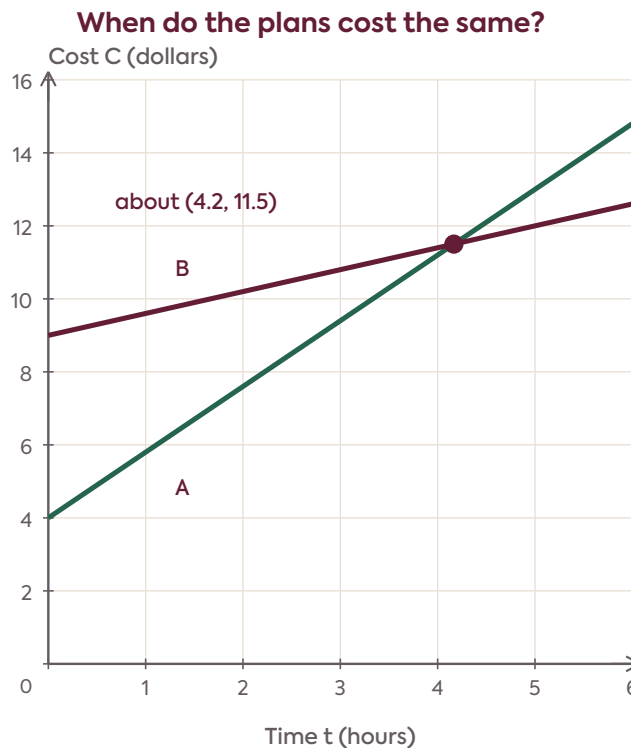
LESSON 16 | A BREAK-EVEN EXAMPLE

An intersection can be an estimate

Two fictional equipment-rental plans charge by time used. With t in hours and C in dollars:

Plan A: $C = 1.8t + 4$; **Plan B:** $C = 0.6t + 9$.

Assume time can include fractions of an hour; both models apply for $0 \leq t \leq 6$.



Read the graph as **about 4.2 hours and \$11.50**. The coordinates mean equal cost at the same rental time. Before the crossing, A costs less; after it, B costs less in this interval.

Use $\approx$ when the graph gives an estimate

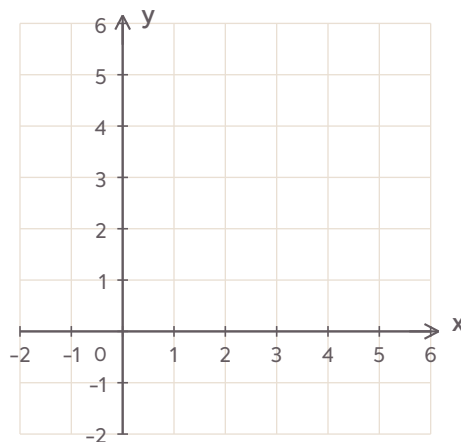
At $t = 4.2$, A gives \$11.56 and B gives \$11.52: close, but not identical because t was rounded. A graph estimate is not an exact algebraic solution. The next two lessons develop exact methods.

LESSON 16 | SHORT CHECKS

Your turn

1. Graph $y = x - 1$ and $y = -x + 3$. Give the intersection, then check it in both equations.

Two lines on the same axes



2. For $x = 0, 1, 2$, one linear table has $y = 2, 5, 8$; the other has $y = 4, 6, 8$. Write the system and its shared point.

3. Classify each system: (a) $y = 2x + 1$ and $y = 2x - 3$; (b) $y = 3x - 2$ and $2y = 6x - 4$.

4. Plan P charges \$20 plus \$3 per visit. Plan Q charges \$5 per visit. Define variables and write two equations. Check whether $(10, 50)$ is their break-even point.

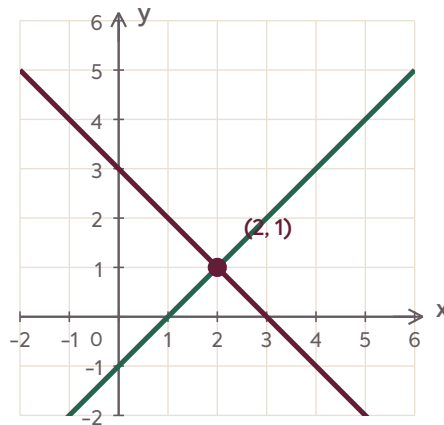
5. A graph suggests an intersection at $(1.7, 4.3)$. Why should you use $\approx$ until you verify both equations?

LESSON 16 | WORKED ANSWERS

Check both relationships

1. The intersection is **(2, 1)**. Both $1 = 2 - 1$ and $1 = -2 + 3$ are true.

Answer 1: intersection (2, 1)



2. $y = 3x + 2$ and $y = 2x + 4$. The shared point is **(2, 8)**: $3(2) + 2 = 8$ and $2(2) + 4 = 8$.

3. **(a) No solution:** same slope, different intercepts. **(b) Infinitely many:** dividing the second equation by 2 gives the first. Solutions lie on $y = 3x - 2$.

4. Let v be visits and C be dollars. $C = 20 + 3v$; $C = 5v$. At $v = 10$, both give \$50, so **(10, 50)** is the break-even point. Here v is a nonnegative whole number.

5. The grid may only support an approximate reading. Rounded coordinates can be close to both lines without satisfying both equations exactly.

I can...

- ☐ Write one equation for each relationship.
- ☐ Interpret and check an intersection.

Next class: substitution finds exact coordinates without relying on a graph. Curriculum focus: A.2I, A.3F, A.3G.

LESSON 17 | MEET THE IDEA

Solving systems by substitution

Tuesday, December 15, 2026 . 6:00–7:00 PM Central

By the end, I can...

Replace a variable with an equal expression, solve the resulting equation and check the ordered pair in both original equations.

Substitution means replacing something with an equal value or expression. If $y = 2x + 1$, then $2x + 1$ can take the place of y in the other equation.

Example 1 | $y = 2x + 1$ and $3x + y = 11$

$$3x + (2x + 1) = 11$$

Replace y in the second equation by its equal expression.

$$5x + 1 = 11$$

Combine the x terms.

$$5x = 10, \text{ so } x = 2$$

Subtract 1, then divide by 5.

$$y = 2(2) + 1 = 5$$

Use $x = 2$ to find the other coordinate.

The solution is $(2, 5)$. Check: $5 = 2(2) + 1$ and $3(2) + 5 = 11$.

Choose the easiest entry point

Look for a variable already alone, or one with coefficient 1 or -1 that is easy to isolate. Either variable can work. Keep the first equation available so you can find the missing coordinate.

LESSON 17 | PARENTHESES PROTECT THE MEANING

Replace the whole expression

Example 2 | $x = 2y - 1$ and $3x - 2y = 9$

$$3(2y - 1) - 2y = 9$$

Replace the entire x with $(2y - 1)$.

$$6y - 3 - 2y = 9$$

Distribute 3 to both terms.

$$4y - 3 = 9$$

Combine like terms.

$$4y = 12, \text{ so } y = 3$$

Add 3, then divide by 4.

$$x = 2(3) - 1 = 5$$

Substitute back to find x .

Solution: $(5, 3)$. Checks: $5 = 2(3) - 1$, and $3(5) - 2(3) = 15 - 6 = 9$.

Spot the error

Incorrect: $3(2y - 1) = 6y - 1$.

Correct: $3(2y - 1) = 6y - 3$.

The multiplier acts on every term inside the parentheses.

What if neither variable is alone?

For $x + y = 10$, subtract x to get $y = 10 - x$. Now substitute $10 - x$ wherever y appears in the other equation. Rewriting an equation first is part of the method.

LESSON 17 | TICKET SALES

Make the answer fit the story

A school sells 18 tickets. Adult tickets cost \$8 and student tickets cost \$5. Sales total \$108. Let a be adult tickets and s be student tickets; both must be nonnegative whole numbers.

Relationship	Equation
18 tickets in total	$a + s = 18$
\$108 in sales	$8a + 5s = 108$

Use the ticket count to isolate s

$s = 18 - a$

$8a + 5(18 - a) = 108$

$8a + 90 - 5a = 108$

$3a = 18$, so $a = 6$

$s = 18 - 6 = 12$

Solve the simpler equation for s .

Replace s in the money equation.

Distribute 5, including its effect on $-a$.

Combine like terms, subtract 90 and divide by 3.

Find the number of student tickets.

Answer and check

6 adult tickets and 12 student tickets.

Count: $6 + 12 = 18$.

Sales: $8(6) + 5(12) = 48 + 60 = \108 .

Both counts are whole numbers and satisfy both conditions.

LESSON 17 | TWO SPECIAL OUTCOMES

When the variables disappear

Substitution does not always leave an equation with one numerical answer. Sometimes the variables cancel. Read what remains.

Same line | $y = 2x + 1$ and $2y = 4x + 2$

$$2(2x + 1) = 4x + 2$$

Substitute for y .

$$4x + 2 = 4x + 2$$

Both sides are the same expression.

$$2 = 2$$

A true statement remains for every x .

There are infinitely many solutions: all points on $y = 2x + 1$. It does not mean every point in the plane is a solution.

Parallel lines | $y = 2x + 1$ and $2y = 4x + 6$

$$2(2x + 1) = 4x + 6$$

Substitute for y .

$$4x + 2 = 4x + 6$$

Distribute.

$$2 = 6$$

A false statement remains. No x can fix it.

There is no solution: the two original equations cannot be true together.

Do not turn $0 = 0$ into $x = 0$

An identity such as $0 = 0$ places no new restriction on x . Use the original equation to describe the shared line. A contradiction such as $0 = 4$ means no shared point.

LESSON 17 | SHORT CHECKS

Your turn

Show your substitution with parentheses, give an ordered pair and check both original equations.

1. Solve $y = x + 2$ and $x + y = 10$.

2. Solve $x = 3y - 2$ and $2x + y = 17$.

3. There are 15 tickets: adult tickets cost \$8, student tickets cost \$5, and sales total \$96. Define variables, write the system and solve.

4. How many solutions does $y = 3x - 2$ with $2y = 6x - 4$ have? Use substitution to explain.

5. For $y = x + 4$ and $3x + 2y = 18$, a student writes $3x + 2x + 4 = 18$. Correct the substitution and solve.

LESSON 17 | WORKED ANSWERS

Check your substitutions

1. $x + (x + 2) = 10$ gives $2x = 8$, so $x = 4$; then $y = 6$. **(4, 6)**. Checks: $6 = 4 + 2$ and $4 + 6 = 10$.
2. $2(3y - 2) + y = 17$ gives $7y - 4 = 17$, so $y = 3$; $x = 7$. **(7, 3)**. Checks: $7 = 3(3) - 2$ and $2(7) + 3 = 17$.
3. Let a and s count adult and student tickets. $a + s = 15$ and $8a + 5s = 96$. Substitute $s = 15 - a$: $8a + 5(15 - a) = 96$, so $3a = 21$. **7 adult tickets and 8 student tickets**. Checks: $7 + 8 = 15$ and $8(7) + 5(8) = 96$.
4. $2(3x - 2) = 6x - 4$ gives $6x - 4 = 6x - 4$, a true statement for every x . **Infinitely many solutions, all on $y = 3x - 2$** .
5. Use **$3x + 2(x + 4) = 18$** . Then $5x + 8 = 18$, so $x = 2$ and $y = 6$. **(2, 6)**. Checks: $6 = 2 + 4$ and $3(2) + 2(6) = 18$. The student failed to multiply 4 by 2.

One equation is not enough for the final check

Back-substitution finds the second coordinate, but only checking both original equations confirms that your pair solves the system.

I can...

- ☐ Choose a useful variable to isolate.
- ☐ Substitute, solve and check both equations.
- ☐ Recognize an identity or contradiction.

Next class: elimination combines equations to remove one variable. Curriculum focus: A.5C.

LESSON 18 | MEET THE IDEA

Solving systems by elimination

Thursday, December 17, 2026 . 6:00–7:00 PM Central

By the end, I can...

Add equivalent equations to eliminate a variable, then find and check both coordinates.

Elimination uses opposite coefficients so one variable adds to zero. Adding equal quantities to equal quantities keeps an equality true.

Example 1 | $3x + 2y = 16$ and $x - 2y = 0$

$$\begin{aligned}(3x + 2y) + (x - 2y) \\ = 16 + 0\end{aligned}$$

Add the entire left sides and the entire right sides.

$$4x = 16$$

The y terms cancel: $2y + (-2y) = 0$.

$$x = 4$$

Divide by 4.

$$4 - 2y = 0, \text{ so } y = 2$$

Use an original equation to find y.

Solution: (4, 2). Checks: $3(4) + 2(2) = 16$; $4 - 2(2) = 0$.

Keep one original equation

The added equation alone is not the whole system. Use it together with an original equation to recover the second coordinate. This is why elimination preserves the shared solution.

LESSON 18 | SCALING ONE OR BOTH EQUATIONS

Make opposite coefficients

Scale one | $2x + 3y = 17$ and $4x + y = 19$

$$-4x - 6y = -34$$

$$\begin{aligned} (-4x - 6y) + (4x + y) \\ = -34 + 19 \end{aligned}$$

$$-5y = -15, \text{ so } y = 3$$

$$4x + 3 = 19, \text{ so } x = 4$$

Multiply every term of the first equation by -2 .

Add it to the unchanged second equation.

The x terms disappear; divide by -5 .

Substitute $y = 3$. Check $(4, 3)$ in both originals.

Scale both | $2x + 3y = 12$ and $3x - 2y = 5$

$$4x + 6y = 24$$

$$9x - 6y = 15$$

$$13x = 39, \text{ so } x = 3$$

$$6 + 3y = 12, \text{ so } y = 2$$

Check $(3, 2)$: $2(3) + 3(2) = 12$ and $3(3) - 2(2) = 5$.

Multiply the first equation by 2 .

Multiply the second equation by 3 .

Add; opposite y terms cancel.

Use the first original equation.

Use a nonzero multiplier and multiply **every term on both sides**. Multiplying only the x term creates a different equation.

LESSON 18 | COUNT AND COST

Use the method in context

A group buys 13 snacks: granola bars at \$3 each and smoothies at \$5 each. They spend \$49. Let g count bars and s count smoothies.

$g + s = 13$ and $3g + 5s = 49$

$-3g - 3s = -39$

$2s = 10$, so $s = 5$

$g + 5 = 13$, so $g = 8$

8 bars and 5 smoothies. Check: $8 + 5 = 13$, and $3(8) + 5(5) = 24 + 25 = \49 .

Multiply the count equation by -3 .

Add to the cost equation: $3g + 5s = 49$.

Use the original count equation.

Which method looks convenient?

What you notice	A useful choice
A variable is already isolated.	Substitution often needs fewer steps.
A pair of coefficients is opposite, or easily made opposite.	Elimination often keeps the arithmetic simple.
You need to see how two quantities compare.	Graphing gives a visual interpretation.

There can be more than one good method

For the snack problem, substitution would also work: $g = 13 - s$. Choose a clear route, and check the same two original conditions at the end.

LESSON 18 | SHORT CHECKS

Your turn

1. Solve $2x + 3y = 13$ and $2x - 3y = 7$.

2. Solve $x + 2y = 8$ and $3x + 2y = 12$. Make opposite coefficients before adding.

3. Solve $2x + 3y = 13$ and $3x - 2y = 0$ by scaling both equations.

4. Nine cartons cost \$46. Small cartons cost \$4; large cartons cost \$6. Write a system and find each count.

5. A student multiplies $2x + 3y = 12$ by -2 and writes $-4x + 3y = -24$. Correct the equation and explain.

Remember the special cases

If valid elimination produces $0 = 0$, use an original equation to describe the shared line. If it produces $0 =$ a nonzero number, there is no solution. The variables disappearing does not mean their values are zero.

LESSON 18 | WORKED ANSWERS

Check, then choose your next step

1. Add: $4x = 20$, so $x = 5$. Then $10 + 3y = 13$ gives $y = 1$. **(5, 1)**. Checks: $10 + 3 = 13$; $10 - 3 = 7$.
2. Multiply the first equation by -1 , then add to the second: $2x = 4$. So $x = 2$ and $y = 3$. **(2, 3)**. Checks: $2 + 6 = 8$; $6 + 6 = 12$.
3. Multiply the first equation by 2 and the second by 3: $4x + 6y = 26$ and $9x - 6y = 0$. Add: $13x = 26$, so $x = 2$; $y = 3$. **(2, 3)**. Checks: $4 + 9 = 13$; $6 - 6 = 0$.
4. Let s and l count small and large cartons. $s + l = 9$; $4s + 6l = 46$. Multiply the first equation by -4 and add: $2l = 10$. **5 large and 4 small cartons**. Checks: $4 + 5 = 9$; $4(4) + 6(5) = 46$.
5. **$-4x - 6y = -24$** . All three terms must be multiplied by -2 , including $3y$. The incorrect equation changes the relationship.

A light plan for the winter break

Complete the normal digital practice set your teacher assigns. For optional retrieval, redo one substitution example and one elimination example without looking at their next lines. Our next live class is **Tuesday, January 5, 2027**.

I can...

- ☐ Create opposite coefficients and eliminate.
- ☐ Check both original equations and explain the context.

Next class: solution regions that satisfy two inequalities together. Curriculum focus: A.5C.

LESSON 19 | MEET THE IDEA

Systems of linear inequalities

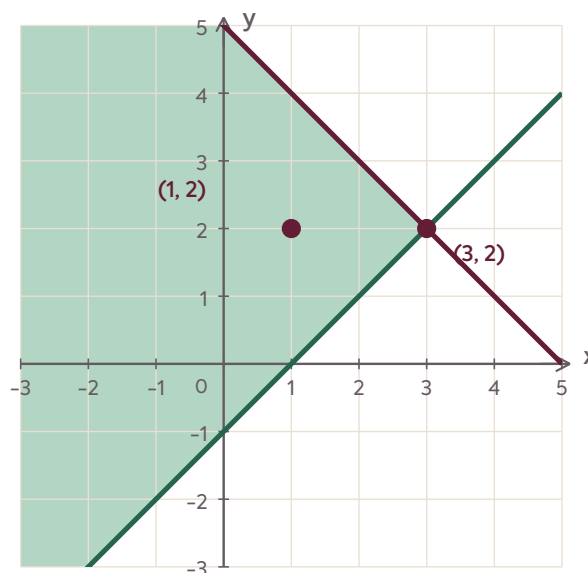
Tuesday, January 5, 2027 . 6:00–7:00 PM Central

By the end, I can...

Graph two linear inequalities on the same axes and find points that satisfy both conditions.

Welcome back. In Class 15, one inequality described one shaded side of a line. Now a **system of inequalities** asks for points that satisfy **both** inequalities. The shared region is the **overlap**.

Both: $y \geq x - 1$ and $y \leq -x + 5$

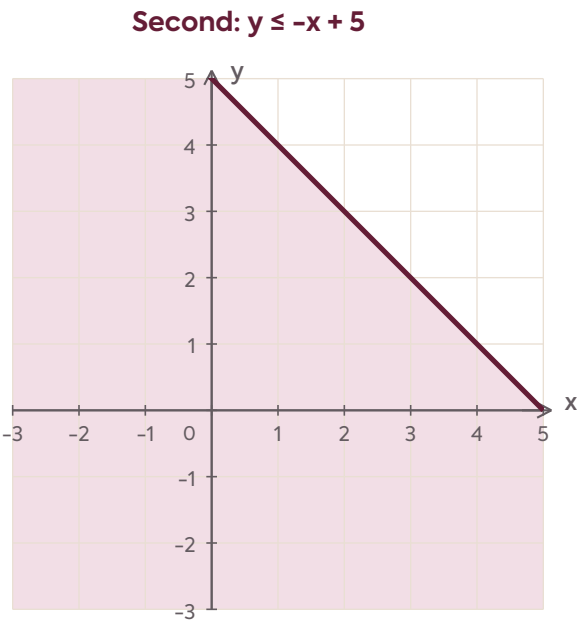
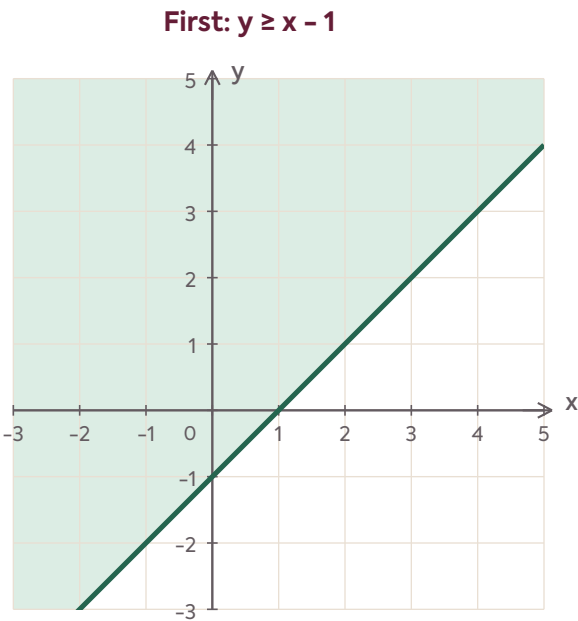


Read AND, not OR

Every point in the shaded overlap works for both rules. A point that works for just one rule is **not** a solution of the system. The displayed region continues to the left beyond this grid.

LESSON 19 | BUILD THE SHARED REGION

Shade each rule, then find the overlap



Use the same scales when combining the graphs. The overlap lies **above the first line and below the second**, including both boundaries wherever the other condition also holds.

Point	$y \geq x - 1$	$y \leq -x + 5$	Both?
(1, 2)	$2 \geq 0$: true	$2 \leq 4$: true	Yes
(3, 0)	$0 \geq 2$: false	$0 \leq 2$: true	No
(3, 3)	$3 \geq 2$: true	$3 \leq 2$: false	No

Boundary points still need both checks

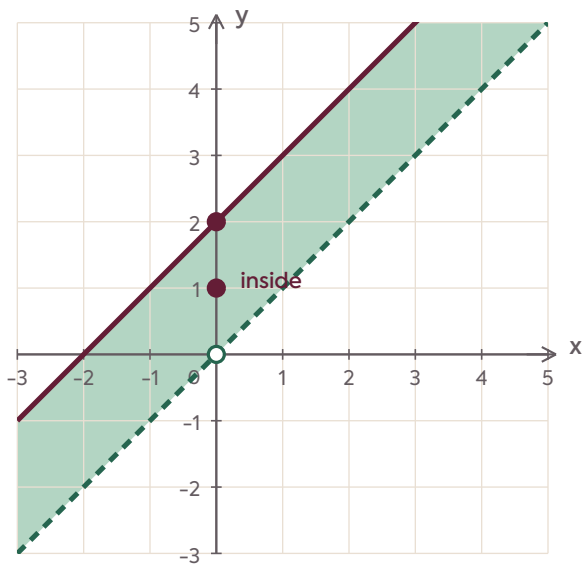
At (3, 2), both equations give $y = 2$, and both inequalities allow equality. This intersection is included. A solid line does not make its entire length a solution of the system.

LESSON 19 | TWO USEFUL CONTRASTS

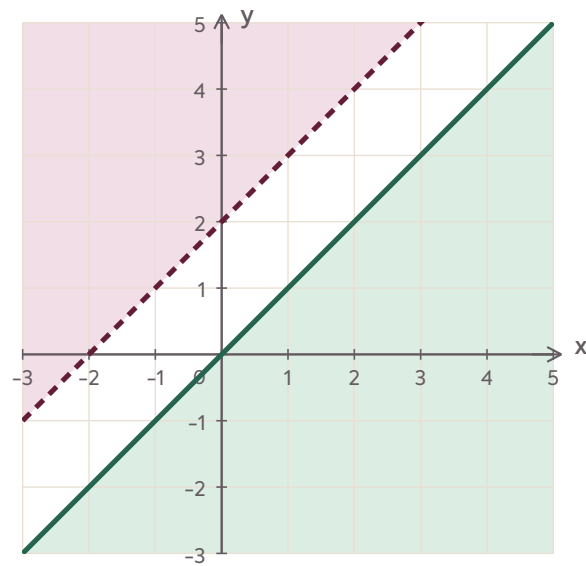
Excluded edges and empty overlap

A strict inequality excludes its boundary. Some pairs of inequalities have no common region at all.

A strip: $y > x$ and $y \leq x + 2$



No overlap: $y > x + 2$; $y \leq x$



Graph	What the region means
Left: a strip	Above $y = x$, excluding that dashed line; at or below $y = x + 2$, including that solid line. $(0, 1)$ and $(0, 2)$ work; $(0, 0)$ does not.
Right: no solution	Pink is above $y = x + 2$; green is at or below $y = x$. These regions never meet: y cannot be greater than $x + 2$ and also no greater than x .

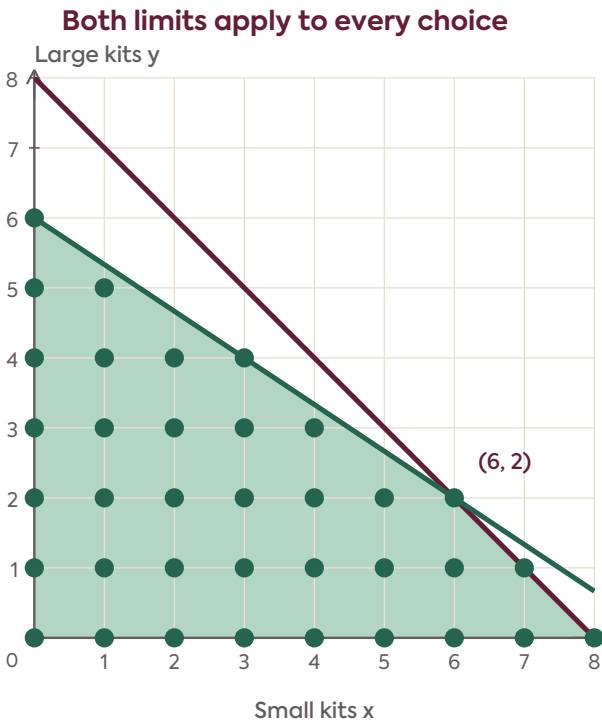
Use the symbols as well as the picture

$\leq$ or $\geq$ includes the boundary. $<$ or $>$ excludes it. A point where a dashed boundary meets another boundary is still excluded if it fails the strict inequality.

LESSON 19 | A FEASIBLE REGION

Find choices that fit two limits

You will make at most 8 craft kits. A small kit needs \$2 of supplies; a large kit needs \$3. Your supplies budget is at most \$18. Let x be small kits and y be large kits.
 $x + y \leq 8$ and $2x + 3y \leq 18$; x and y are nonnegative whole numbers.



The dots are **feasible choices**: choices that obey every restriction. The whole shaded shape models the numerical limits; only its nonnegative whole-number points are allowed kit counts.

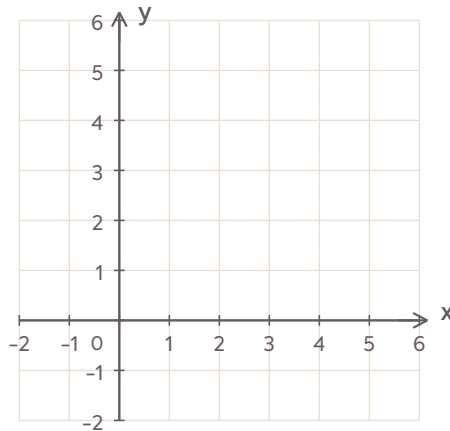
Choice	Number of kits	Supplies cost	Feasible?
(2, 4)	$6 \leq 8$	$\$16 \leq \18	Yes
(4, 4)	$8 \leq 8$	$\$20 > \18	No
(6, 2)	$8 \leq 8$	$\$18 \leq \18	Yes

LESSON 19 | SHORT CHECKS

Your turn

1. Graph $y \geq 0$ and $y < -x + 4$ together. Shade only the overlap and show included/excluded boundaries.

Your shared solution region



2. For $y \geq x - 1$ and $y \leq -x + 5$, test $(2, 2)$, $(4, 2)$ and $(0, -2)$ in both inequalities.

3. Does $y > x + 3$ together with $y \leq x + 1$ have any solution? Explain.

4. In the craft-kit example, is $(5, 3)$ feasible? Check the count and cost limits.

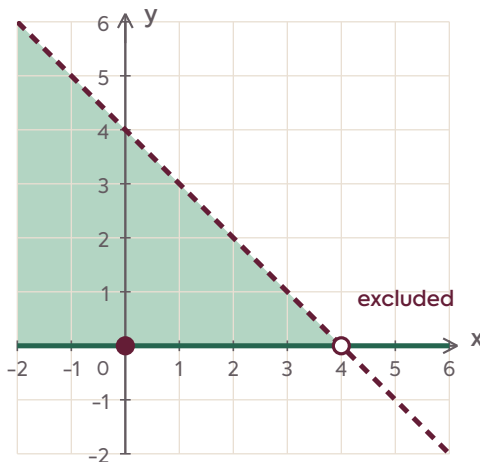
5. For $x \geq 1$ and $y > 2$, a student shades the entire plane to the right of $x = 1$. What is missing? Describe the correct overlap.

LESSON 19 | WORKED ANSWERS

Check every condition

1. Use $y = 0$ **solid** and $y = -x + 4$ **dashed**. Shade at or above the x-axis and strictly below the sloping line. The meeting point $(4, 0)$ is excluded because $0 < 0$ is false.

Answer 1: $y \geq 0$ AND $y < -x + 4$



2. **(2, 2): yes** ($2 \geq 1$ and $2 \leq 3$). **(4, 2): no** ($2 \geq 3$ is false and $2 \leq 1$ is false). **(0, -2): no** ($-2 \geq -1$ is false, even though $-2 \leq 5$ is true).

3. **No solution.** A number greater than $x + 3$ cannot also be at most the smaller number $x + 1$. There is no overlap.

4. **No.** There are $5 + 3 = 8$ kits, but supplies cost $2(5) + 3(3) = \$19$, which exceeds \$18.

5. The second condition is missing. Shade **right of or on $x = 1$ AND above $y = 2$** . The vertical boundary is solid; the horizontal boundary is dashed.

I can...

- ☐ Find overlap and check both inequalities.
- ☐ Respect strict boundaries and contextual restrictions.

Next class: review and the Chapter 3 checkpoint. Curriculum focus: A.3H.

LESSON 20 | MEET THE IDEA

Unit 3 checkpoint and systems in context

Thursday, January 7, 2027 . 6:00–7:00 PM Central

By the end, I can...

Choose a method, explain a solution in context and use feedback to repair a specific skill.

This lesson brings together the chapter. These pages are your **revision and correction guide**. Your teacher supplies the graded checkpoint separately; its questions and key are not in this PDF.

If the task asks you to...	A useful route
Find where two quantities match	Write two equations; graph or solve the system.
Solve with one variable already isolated	Use substitution with parentheses.
Solve with matching or opposite coefficients	Use elimination, scaling whole equations if needed.
Find choices that obey two limits	Write inequalities; graph and test the overlap.

Our one-hour checkpoint lesson

5 minutes: recall key ideas.

20 minutes: the chapter checkpoint.

25 minutes: corrections and targeted teaching.

10 minutes: exit check and next steps.

There is one checkpoint, not a second separate quiz.

LESSON 20 | A METHOD AND ERROR GUIDE

Use a reason, not a guess

Same answer, two routes | $y = x + 1$ and $x + y = 7$

$x + (x + 1) = 7$

$2x = 6; x = 3, y = 4$

$-x + y = 1$
 $x + y = 7$

$2y = 8; y = 4, x = 3$

Substitution is convenient because y is already isolated.

Find both coordinates: $(3, 4)$.

For elimination, first rewrite the first equation.

Add to eliminate x . Check: $4 = 3 + 1; 3 + 4 = 7$.

Mistake	Repair
Only checking one original equation	Substitute the ordered pair into both.
$2(x + 3)$ becomes $2x + 3$	Distribute to both terms: $2x + 6$.
Scaling only one side or term	Multiply every term on both sides.
Shading every point in either region	Keep only the overlap for an AND system.
Including a strict boundary	Use a dashed line; equality does not satisfy $<$ or $>$.

LESSON 20 | PRACTICE, NOT THE GRADED CHECKPOINT

A short revision check

1. Solve $y = 2x - 1$ and $x + y = 8$. State why your chosen method is convenient.
-
-
2. A club sells 10 tickets for \$64. Adult tickets are \$8 and student tickets \$4. Write and solve a system, then check both totals.
-
-
3. For $y \geq x$ and $y < -x + 4$, check whether (1, 2) and (2, 2) belong. Which boundaries are included?
-
-
4. Classify: (a) $2x + y = 6$ and $4x + 2y = 12$; (b) $2x + y = 6$ and $4x + 2y = 14$.
-
-

My correction organizer

Write one specific thing	My notes
The first step I need to repair	
The earlier lesson I will revisit	
How I will check a fresh question	

Revisit Classes 15 and 19 for inequalities; Class 16 for writing and graphing systems; Class 17 for substitution; Class 18 for elimination.

LESSON 20 | WORKED ANSWERS AND NEXT STEPS

Check, correct and move forward

1. Substitution is convenient: $x + (2x - 1) = 8$ gives $3x = 9$, so $x = 3$ and $y = 5$. **(3, 5)**. Checks: $5 = 2(3) - 1$ and $3 + 5 = 8$.
2. Let a and s count adult and student tickets. $a + s = 10$; $8a + 4s = 64$. Multiply the first by -4 and add: $4a = 24$. **6 adult tickets and 4 student tickets**. Checks: $6 + 4 = 10$; $8(6) + 4(4) = \$64$.
3. **(1, 2) belongs**: $2 \geq 1$ and $2 < 3$ are true. **(2, 2) does not**: $2 \geq 2$ is true, but $2 < 2$ is false. The boundary $y = x$ is solid and $y = -x + 4$ is dashed. Only boundary portions satisfying both conditions belong.
4. **(a) Infinitely many solutions**: the second equation is twice the first, so both describe $2x + y = 6$. **(b) No solution**: doubling the first gives $4x + 2y = 12$, which cannot also equal 14.

Use the evidence to choose your next step

A corrected answer is useful when you can explain the changed step. Revisit that example, cover the solution and try a fresh question. Your teacher will use class work, practice and the checkpoint together to decide what needs more attention.

Ready for the next chapter

- ☐ I can model a situation with two equations or inequalities.
- ☐ I can solve or graph, and check every condition.
- ☐ I can name one skill to strengthen and how I will practice it.

Chapter 4 starts Tuesday, January 12, 2027: exponents, radicals and polynomial operations. Curriculum focus: A.2H-I, A.3D and A.3F-H, A.5C.

KEEP THIS PAGE FOR REVISION

Chapter 3 quick reference

Idea	Remember	Example
Boundary	Solid for $\leq$ or $\geq$. Dashed for $<$ or $>$.	$y < x + 2$ excludes $y = x + 2$.
Shaded side	Test a point off the boundary in the original inequality.	$(0, 0)$ satisfies $y \leq -x + 4$.
Negative divisor	Reverse the inequality when multiplying or dividing by a negative number.	$-2y \leq 4 - x$ becomes $y \geq (1/2)x - 2$.
Equation system	A solution satisfies both equations. On a graph, it is a shared point.	$y = x + 1$ and $y = -x + 5$ meet at $(2, 3)$.
Substitution	Replace a variable with an equal expression; use parentheses.	$y = x + 2$ in $x + y = 10$ gives $x + (x + 2) = 10$.
Elimination	Scale whole equations by nonzero numbers, then add opposite terms.	$2y + (-2y) = 0$. Find the remaining coordinate afterward.
Special cases	Coincident lines: infinitely many. Distinct parallel lines: none.	An identity such as $0 = 0$ differs from $0 = 4$.
Inequality system	Keep only the overlap; each point must obey every condition.	At or above $y = x$ and below $y = -x + 4$.
Context	Define units and sensible restrictions. Check the original story.	Counts of kits must be nonnegative whole numbers.

Graph readings may be estimates. Use $\approx$ when coordinates are approximate, and substitute into both equations to verify an exact intersection.

STUDY GUIDANCE AND CURRICULUM REFERENCES

Make these notes work for you

A small routine for each lesson

Before: recall the earlier skill named in the lesson.

During: cover the next worked step, predict it and explain why it works.

After: try the short checks before looking at their answers, then complete the assigned digital practice.

Study notes and assessment

These PDFs contain worked answers to their own short practice checks. The Class 16 quiz and Class 20 graded checkpoint are separate teacher-provided materials; their unpublished questions and keys are not included.

Graphs that communicate clearly

Label your axes, units and scales. Identify each boundary and whether it is included. A graph window shows only part of an unbounded region. For a context involving counts, select whole-number points as well as checking the inequalities. All stories here are fictional teaching examples.

Curriculum alignment

Standard Algebra I, §111.39(c): A.2H–I; A.3D and A.3F–H; A.5C. Advanced Grade 8 Algebra I, §111.31(d): (2)(H)–(I), (3)(G) and (I)–(K), and (5)(C). Mathematical processes and earlier linear skills are revisited throughout. The two pathways use distinct codes.

These original Cambridge Veritas supplementary tutoring notes follow the agreed teaching sequence. They are not an official TEA publication. The dates are the Cambridge Veritas class calendar, not a state pacing requirement.

Official references

Texas Education Agency: [Current mathematics standards index](#).

Standard Algebra I: [§111.39, high-school mathematics TEKS](#).

Advanced Grade 8 Algebra I: [§111.31, middle-school mathematics TEKS, adopted 2025](#).

Standards checked September 9, 2026. Chapter 3 edition: 1.0.