



CHAPTER 05

Algebra I

Factoring and quadratic functions

Classes 27–32 . February 2–18, 2027
Tuesdays and Thursdays . 6:00–7:00 PM Central

ONE FUNCTION, THREE USEFUL FORMS

$$y = x^2 - 2x - 3$$

Standard form shows the coefficients.

$$y = (x - 3)(x + 1)$$

Factored form shows a product.

$$y = (x - 1)^2 - 4$$

Vertex form shows the turning point.

Clear explanations . Worked examples . Accurate graphs
Short practice checks . Worked answers . Chapter revision

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Your route through Chapter 5

Learn one lesson at a time, then return to the examples when you revise. The short study checks have worked answers at the end of each lesson.

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Chapter quick reference: page 38 . Study guidance and sources: page 39

Use these notes during the 60-minute lessons and for revision afterward. Your teacher assigns digital homework separately.

LESSON 27 | MEET THE IDEA

Common factors and monic quadratic trinomials

Tuesday, February 2, 2027 . 6:00–7:00 PM Central

By the end, I can...

Rewrite a sum as a product, take out a greatest common factor, and factor a monic quadratic using a product-and-sum check.

Factoring reverses multiplication. Expanding turns a product into a sum. Factoring turns that sum back into a product with the same value.

Read the distributive property in both directions

$$3x(x + 4) = 3x^2 + 12x$$

Expand: multiply $3x$ by each term in the parentheses.

$$3x^2 + 12x = 3x(x + 4)$$

Factor: both terms contain $3x$. Divide each term by that common factor.

Find the greatest common factor (GCF)

Use the greatest common numerical factor and the lowest exponent of every variable shared by **all** terms.

Factor $12x^3 + 18x^2$

$$\text{GCF} = 6x^2$$

12 and 18 share 6; x^3 and x^2 share two factors of x .

$$6x^2(2x + 3)$$

$$12x^3 \div 6x^2 = 2x; 18x^2 \div 6x^2 = 3.$$

Check by expansion: $6x^2(2x) + 6x^2(3) = 12x^3 + 18x^2$. The factored identity is valid for every real x , including 0.

LESSON 27 | MONIC QUADRATIC TRINOMIALS

Two numbers do two jobs

A **quadratic trinomial** has three terms and degree 2. **Monic** means the coefficient of x^2 is 1.

Why the product-and-sum method works

$(x + m)(x + n)$	Multiply every term by every term.
$x^2 + nx + mx + mn$	The middle two terms are like terms.
$x^2 + (m + n)x + mn$	The sum $m + n$ is the x -coefficient; the product mn is the constant.

For $x^2 + bx + c$

Find numbers m and n with $mn = c$ and $m + n = b$.
Then $x^2 + bx + c = (x + m)(x + n)$.

Factor pair for 12	Sum	Does it give 7?
1 and 12	13	No
2 and 6	8	No
3 and 4	7	Yes

So $x^2 + 7x + 12 = (x + 3)(x + 4)$.
Check: $x^2 + 4x + 3x + 12 = x^2 + 7x + 12$.

LESSON 27 | NEGATIVE TERMS AND GCF FIRST

Let the signs guide your search

Constant c	What to look for
Positive	Same signs. A positive sum needs two positives; a negative sum needs two negatives.
Negative	Opposite signs. Their sum must still equal the x-coefficient.

Factor $x^2 - x - 12$

$m \times n = -12; m + n = -1$
 $m = -4; n = 3$
 $(x - 4)(x + 3)$

The numbers must have opposite signs.

$(-4)(3) = -12$ and $-4 + 3 = -1$.

Check: $x^2 + 3x - 4x - 12 = x^2 - x - 12$.

Factor $2x^2 - 10x + 12$ completely

$2(x^2 - 5x + 6)$
 $2(x - 2)(x - 3)$

Take out the GCF, 2, before using the monic method.

The pair -2 and -3 has product 6 and sum -5.

Check: $2(x^2 - 5x + 6) = 2x^2 - 10x + 12$. Keep the factor 2 in the final answer.

Factor an expression; do not invent an equation

An expression such as $x^2 - x - 12$ has no equals sign. The task is to rewrite it, not to find x. Solving a quadratic equation comes in Chapter 6.

LESSON 27 | SHORT CHECKS

Your turn

Factor each expression completely over the integers. Check by expanding. For Question 6, explain the error in words.

1. $8x^2 + 12x$

2. $x^2 + 9x + 20$

3. $x^2 - 7x + 10$

4. $x^2 + 2x - 15$

5. $3x^2 - 3x - 18$

6. Sam says $x^2 + 6x + 8 = (x + 1)(x + 8)$ because $1 \times 8 = 8$. What did Sam miss?

LESSON 27 | WORKED ANSWERS

Check both the product and the sum

1. $4x(2x + 3)$. The GCF is $4x$; expanding gives $8x^2 + 12x$.
2. $(x + 4)(x + 5)$. $4 \times 5 = 20$ and $4 + 5 = 9$. The cross-products give $5x + 4x = 9x$.
3. $(x - 5)(x - 2)$. $(-5)(-2) = 10$ and $-5 + (-2) = -7$. Expanding gives $x^2 - 7x + 10$.
4. $(x + 5)(x - 3)$. $5(-3) = -15$ and $5 - 3 = 2$. The two signs must be different.
5. $3(x^2 - x - 6) = 3(x - 3)(x + 2)$. Keep the GCF 3. Expansion gives $3x^2 - 3x - 18$.
6. Sam did not check the **sum**: $1 + 8 = 9$, not 6. Use 2 and 4, so the correct factors are $(x + 2)(x + 4)$.

If no integer pair works

Do not force an answer. For $x^2 + x - 1$, no integer pair has product -1 and sum 1 . This does **not** prove that no real factors exist. Later lessons explain non-integer real factors.

I can...

- ☐ Take out a GCF before factoring further.
- ☐ Check the product, sum and signs of a factor pair.
- ☐ Verify a factorization by expanding.

Next class: quadratics whose leading coefficient is not 1. Curriculum focus: A.10D, A.10E.

LESSON 28 | MEET THE IDEA

Nonmonic and perfect-square trinomials

Thursday, February 4, 2027 . 6:00–7:00 PM Central

By the end, I can...

Factor a nonmonic quadratic using one reliable grouping method, and recognize a perfect-square trinomial by checking all three terms.

In a **nonmonic** quadratic $ax^2 + bx + c$, the leading coefficient a is not 1. After taking out any GCF, we will **split the middle term and group**.

One method: multiply, split, group, check

1. Calculate ac .
2. Find two integers with product ac and sum b .
3. Replace bx with those two x -terms.
4. Factor the two groups, then their common binomial.
5. Expand to check.

Factor $6x^2 + 11x + 3$

$$ac = 6 \times 3 = 18$$

$$6x^2 + 9x + 2x + 3$$

$$3x(2x + 3) + 1(2x + 3)$$

$$(3x + 1)(2x + 3)$$

Find a pair with product 18 and sum 11: 9 and 2.

Splitting $11x$ into $9x + 2x$ keeps the expression equivalent.

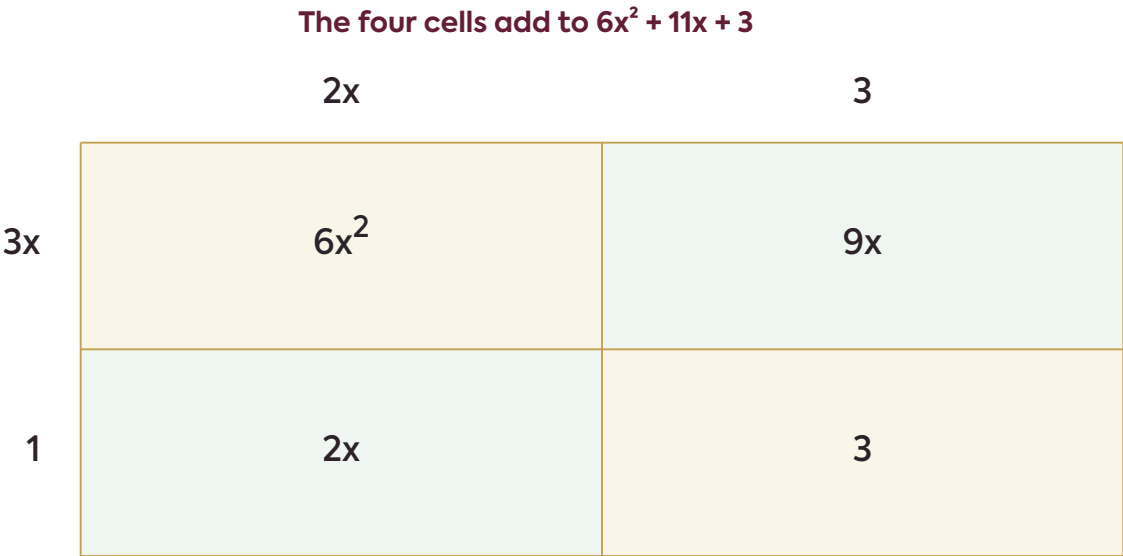
Factor each group. The binomials match.

Factor out the shared binomial, $2x + 3$.

Notice: 9 and 2 split the middle term. They are not automatically the constants in the final factors.

LESSON 28 | THE SAME EXAMPLE, ORGANIZED VISUALLY

See the groups in a grid



This is an algebra grid, not a scale drawing.

Each cell is its row label multiplied by its column label. Add the row labels to get $3x + 1$; add the column labels to get $2x + 3$. The grid records the same grouping method.

Multiply back to verify

$(3x + 1)(2x + 3)$

$6x^2 + 9x + 2x + 3$

$6x^2 + 11x + 3$

Every term meets every term.

The middle two products add to $11x$.

The original trinomial is recovered.

When the groups do not match

Recheck the pair, signs and GCFs. You may reorder the two split terms. Do not drop a term or change a sign just to make matching parentheses.

LESSON 28 | KEEP EVERY SIGN

Use grouping with negative terms

Factor $6x^2 - x - 2$

$$ac = -12; b = -1$$

$$6x^2 - 4x + 3x - 2$$

$$2x(3x - 2) + 1(3x - 2)$$

$$(2x + 1)(3x - 2)$$

Use -4 and 3: their product is -12 and their sum is -1.

Split the middle term into $-4x + 3x$.

Factor each pair.

Check: $6x^2 - 4x + 3x - 2 = 6x^2 - x - 2$.

Factor $2x^2 - 7x + 3$

$$ac = 6; b = -7$$

$$2x^2 - 6x - x + 3$$

$$2x(x - 3) - 1(x - 3)$$

$$(2x - 1)(x - 3)$$

Use -6 and -1: same signs, negative sum.

Split $-7x$ into $-6x - x$.

Take out -1 from the second group: $-x + 3 = -1(x - 3)$.

Check: $2x^2 - 6x - x + 3 = 2x^2 - 7x + 3$.

LESSON 28 | CHECK THE MIDDLE TERM

A trinomial that is a square

A **perfect-square trinomial** is the product of a binomial with itself. The first and last terms being squares is not enough: the middle term must also match.

The two patterns

$$u^2 + 2uv + v^2 = (u + v)^2$$

$$u^2 - 2uv + v^2 = (u - v)^2$$

Factor $9x^2 - 24x + 16$

$$u = 3x; v = 4$$

$$-2(3x)(4) = -24x$$

$$(3x - 4)^2$$

$$(3x)^2 = 9x^2 \text{ and } 4^2 = 16.$$

The middle term matches the minus pattern.

Check by expanding $(3x - 4)(3x - 4)$.

Trinomial	Decision
$x^2 + 10x + 25$	$(x + 5)^2$, since $2(x)(5) = 10x$.
$4x^2 + 12x + 9$	$(2x + 3)^2$, since $2(2x)(3) = 12x$.
$4x^2 + 10x + 9$	Not $(2x + 3)^2$: that expansion has $12x$, not $10x$.

Always look for a GCF first: $2x^2 + 12x + 18 = 2(x^2 + 6x + 9) = 2(x + 3)^2$.

LESSON 28 | SHORT CHECKS

Your turn

Use grouping for Questions 1–3. Factor completely in Questions 4–5. Explain your decision in Question 6.

1. $2x^2 + 7x + 3$

2. $3x^2 - 5x - 2$

3. $6x^2 - 7x + 2$

4. $4x^2 - 20x + 25$

5. $2x^2 + 12x + 18$

6. Is $9x^2 + 12x + 4$ equal to $(3x + 2)^2$? What about $9x^2 + 10x + 4$?

LESSON 28 | WORKED ANSWERS

Check the middle term every time

1. $ac = 6$; split $7x$ as $6x + x$.

$$2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3).$$

2. $ac = -6$; split $-5x$ as $-6x + x$.

$$3x^2 - 6x + x - 2 = 3x(x - 2) + 1(x - 2) = (3x + 1)(x - 2).$$

3. $ac = 12$; split $-7x$ as $-4x - 3x$.

$$6x^2 - 4x - 3x + 2 = 2x(3x - 2) - 1(3x - 2) = (2x - 1)(3x - 2).$$

4. $(2x - 5)^2$. The middle term is $-2(2x)(5) = -20x$; the last term is 25.

5. Take out 2 first: $2(x^2 + 6x + 9) = 2(x + 3)^2$.

6. **Yes** for $9x^2 + 12x + 4$ because $2(3x)(2) = 12x$. **No** for $9x^2 + 10x + 4$: its middle term does not match the claimed square.

I can...

- ☐ Use the ac pair to split the middle term.
- ☐ Factor a negative group without losing a sign.
- ☐ Recognize a square by checking its middle term.

Next class: differences of squares and selecting a factoring method. Curriculum focus: A.10E.

LESSON 29 | MEET THE IDEA

Differences of squares and choosing a factoring method

Tuesday, February 9, 2027 . 6:00–7:00 PM Central

By the end, I can...

Recognize a difference of two squares, choose a sensible factoring method and explain what an unsuccessful integer-factor search does and does not tell me.

A **difference of squares** is one square subtracted from another. Multiplying a sum and a difference makes the middle terms cancel.

Where the pattern comes from

$$(u - v)(u + v)$$

Multiply each term in the first factor by each in the second.

$$u^2 + uv - uv - v^2$$

The middle terms are opposites.

$$u^2 - v^2$$

The result has two terms.

The pattern

$$u^2 - v^2 = (u - v)(u + v)$$

One factor has a minus sign; the other has a plus sign.

Factor $25x^2 - 16$

$$(5x)^2 - 4^2$$

Identify what is being squared.

$$(5x - 4)(5x + 4)$$

Check: $25x^2 + 20x - 20x - 16 = 25x^2 - 16$.

LESSON 29 | MORE THAN ONE STEP

GCF first; then look again

Factor $12x^2 - 75$ completely

$$3(4x^2 - 25)$$

$$3[(2x)^2 - 5^2]$$

$$3(2x - 5)(2x + 5)$$

Check: $3(4x^2 - 25) = 12x^2 - 75$.

The GCF of the two terms is 3.

The expression inside is a difference of squares.

Keep the GCF outside both binomials.

Factor $9 - x^2$

$$3^2 - x^2$$

$$(3 - x)(3 + x)$$

The first square is 3^2 . The order matters.

Check: $9 + 3x - 3x - x^2 = 9 - x^2$.

A sum of squares is different

$x^2 + 9$ is not $(x + 3)(x - 3)$; that product is $x^2 - 9$.

For real x , $x^2 + 9$ is always positive. It has no real zeros and cannot be written as a product of real linear factors.

Do not insert the difference-of-squares pattern into a sum. The operation sign is part of the pattern, not decoration.

LESSON 29 | A FACTORING DECISION GUIDE

Choose a method with a reason

Step	Look for	Action
1. Always start here	A factor shared by every term	Take out the GCF. If useful, take out a negative factor to leave a positive leading term.
2. Count terms inside	Two terms, with subtraction	Check for a difference of two squares.
3. Three terms	A perfect-square pattern	Check the first, last AND middle terms.
4. Other trinomials	Leading coefficient 1	Use product c and sum b.
5. Other trinomials	Leading coefficient not 1	Use ac, split the middle term and group.
6. Finish	Factors that can still be factored	Continue if possible; expand the full result to check.

No integer pair is not the end of the story

$x^2 - 2 = (x - \sqrt{2})(x + \sqrt{2})$.

The factors use real numbers, but not integers. This is still the same difference-of-squares pattern because $(\sqrt{2})^2 = 2$.

For today, use integer factors unless a question asks for real factors. If an integer search fails, report that precisely. Chapter 6 develops ways to find other real roots and their factors.

LESSON 29 | SHORT CHECKS

Your turn

Factor completely over the integers in Questions 1–5. Follow the separate instructions in Questions 6–7.

1. $x^2 - 49$

2. $4x^2 - 9$

3. $8x^2 - 32$

4. $x^2 + 8x + 16$

5. $2x^2 + 5x + 2$

6. Factor $x^2 - 5$ using real numbers. Check by multiplying.

7. A student writes $x^2 + 16 = (x - 4)(x + 4)$. Explain why it is wrong.

LESSON 29 | WORKED ANSWERS

Check the pattern and the domain

1. $(x - 7)(x + 7)$. The middle terms cancel, leaving $x^2 - 49$.

2. $(2x - 3)(2x + 3)$. The squared terms are $(2x)^2$ and 3^2 .

3. $8(x^2 - 4) = 8(x - 2)(x + 2)$. The GCF 8 stays in the result.

4. $(x + 4)^2$. The middle term is $2(x)(4) = 8x$, so the perfect-square pattern fits.

5. $ac = 4$; split $5x$ as $4x + x$.

$2x(x + 2) + 1(x + 2) = (2x + 1)(x + 2)$.

6. $(x - \sqrt{5})(x + \sqrt{5})$. The product is $x^2 - (\sqrt{5})^2 = x^2 - 5$. These are real linear factors with irrational constants.

7. The proposed product expands to $x^2 - 16$, not $x^2 + 16$. Also, $x^2 + 16 \geq 16$ for real x , so it has no real zeros or real linear factors.

I can...

- ☐ Look for a GCF before another pattern.
- ☐ Distinguish a difference of squares from a sum.
- ☐ State whether I am looking for integer or real factors.

Next class: meet quadratic graphs and their key features. Curriculum focus: A.10D-F.

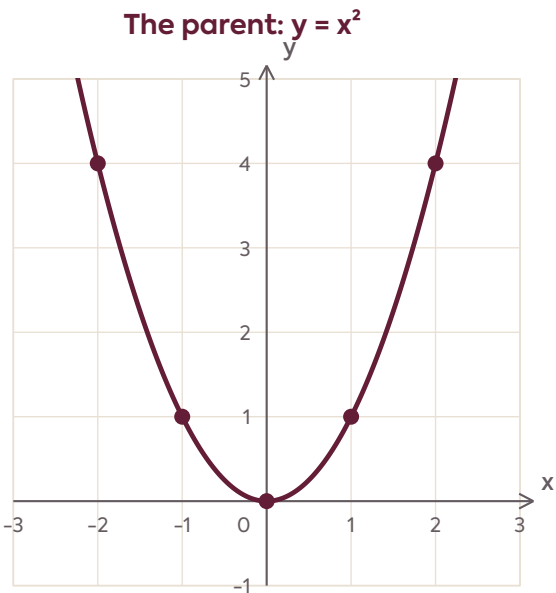
LESSON 30 | MEET THE IDEA

Quadratic graphs and their key features

Thursday, February 11, 2027 . 6:00–7:00 PM Central

By the end, I can...
Read a quadratic graph, identify its key features, and state a sensible domain and range in mathematics and in context.

A **quadratic function** can be written $y = ax^2 + bx + c$, with $a \neq 0$. Its graph is a curved shape called a **parabola**. Begin with $y = x^2$.



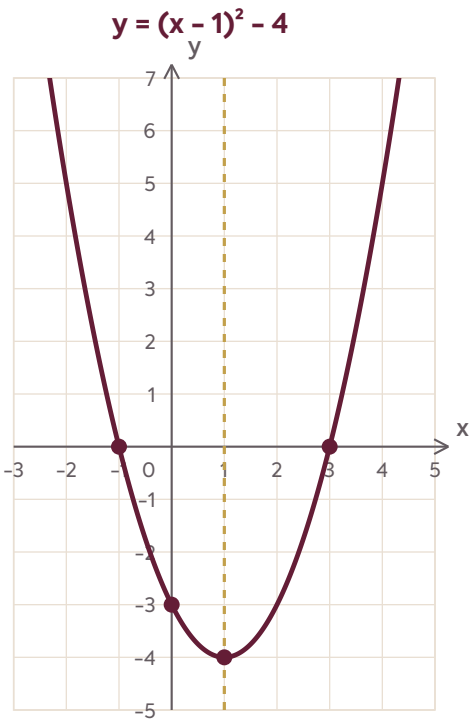
- Vertex: (0, 0)**
The lowest point.
- Axis of symmetry: $x = 0$**
The left and right halves mirror each other.
- Opening: upward**
Both arms continue beyond the window.

x	-2	-1	0	1	2
$y = x^2$	4	1	0	1	4

Plot the table points, then draw a smooth curve through them. Equal distances left and right of the axis give equal y-values. Do not join the points with sharp corners.

LESSON 30 | VERTEX, INTERCEPTS AND SYMMETRY

Read a parabola like a map



- Vertex:** (1, -4)
- Axis:** $x = 1$ (dashed)
- Minimum value:** -4
- x-intercepts:** (-1, 0), (3, 0)
- Zeros:** -1 and 3
- y-intercept:** (0, -3)

The dots mark the vertex and intercepts. The curve continues past the graph window.

Feature	What it tells you
Vertex (h, k)	The turning point. Here it is (1, -4).
Axis $x = h$	A vertical line through the vertex, not a single point.
Zero versus x-intercept	A zero is an x-value, such as 3. The corresponding intercept is a point, (3, 0).
y-intercept	Where $x = 0$. Here $(0 - 1)^2 - 4 = -3$.

LESSON 30 | INPUTS, OUTPUTS AND RESTRICTIONS

Domain and range answer different questions

Domain: which x-values can go in?
Range: which y-values can come out? A graph window shows only part of an unrestricted quadratic.

Function	Domain	Range
$y = (x - 1)^2 - 4$	All real x	$y \geq -4$
$y = -(x + 2)^2 + 5$	All real x	$y \leq 5$
$y = x^2 + 2$	All real x	$y \geq 2$

Why the vertex gives the range
In $y = a(x - h)^2 + k$, the squared part is never negative.
a > 0: the graph opens up; its minimum value is k, so $y \geq k$.
a < 0: it opens down; its maximum value is k, so $y \leq k$.
These range rules assume the full, unrestricted quadratic.

A parabola may have two, one or no real zeros

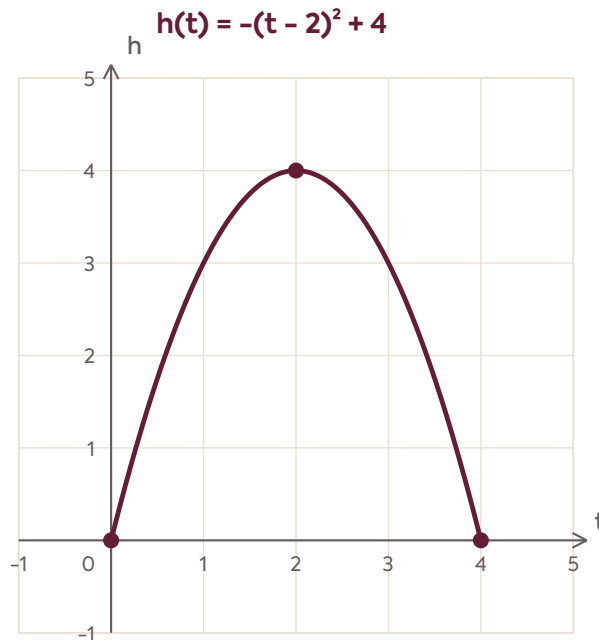
y = x² - 1: crosses the x-axis twice, at x = -1 and x = 1.
y = x²: touches the x-axis at x = 0.
y = x² + 2: never reaches the x-axis; there are no real zeros.

For an unrestricted quadratic, all real x may also be written $-\infty < x < \infty$. Infinity is not a number you substitute into the function.

LESSON 30 | A SIMPLE HEIGHT MODEL

A story can restrict the graph

A fictional model describes an object rising from the ground and returning to it: $h(t) = -(t - 2)^2 + 4$. Time t is in seconds; height h is in meters. Use the model only from launch to return.



Time t : seconds

Height h : meters

Start: $(0, 0)$

Highest point: $(2, 4)$

Return: $(4, 0)$

Only the part from $t = 0$ to $t = 4$ is used in this model.

Read the restrictions from the situation

Domain: $0 \leq t \leq 4$. The modeled time starts at launch and ends on return.

Range: $0 \leq h \leq 4$. The object stays between ground level and its maximum height in this interval.

The full formula would give $h(5) = -5$. That does not mean this story continues five meters below ground: $t = 5$ is outside the chosen model. A formula can be defined beyond the part that makes sense in a situation.

LESSON 30 | READ, CALCULATE AND EXPLAIN

Your turn

Use a table or graphing tool if helpful. State exact values for these examples. These are study checks, not the graded unit checkpoint.

1. For $y = (x - 2)^2 - 1$, make a table for $x = 0, 1, 2, 3, 4$. Identify the vertex, axis, intercepts and zeros.

2. For the unrestricted function in Question 1, state the domain, range and minimum value.

3. For $y = -(x + 1)^2 + 4$, identify the vertex, axis, maximum value, range and y-intercept.

4. Does $y = x^2 + 3$ have real zeros? Explain without trying every x-value.

5. In the height model on the previous page, find the height at $t = 1$ and $t = 3$. Explain the equal heights and why $t = 6$ should not be used.

LESSON 30 | WORKED ANSWERS

Check the feature, not just the number

1. The y-values are **3, 0, -1, 0, 3**. Vertex: **(2, -1)**; axis: **$x = 2$** . The x-intercepts are **(1, 0), (3, 0)**, so the zeros are **1 and 3**. The y-intercept is **(0, 3)**.
2. Domain: **all real x**. The graph opens up and its lowest output is -1, so the range is **$y \geq -1$** and its minimum value is **-1**.
3. Vertex: **(-1, 4)**; axis: **$x = -1$** ; maximum value: **4**. The full graph opens down, so its range is **$y \leq 4$** . At $x = 0$, $y = -(1)^2 + 4 = 3$, giving **(0, 3)**.
4. **No real zeros**. Since $x^2 \geq 0$ for every real x , $x^2 + 3 \geq 3$. The graph stays above the x-axis.
5. $h(1) = -(-1)^2 + 4 = 3$ **meters**; $h(3) = -(1)^2 + 4 = 3$ **meters**. The times are equally far from $t = 2$, the axis of symmetry. **$t = 6$ is outside $0 \leq t \leq 4$** , the chosen time interval.

When using a graphing tool

Enter parentheses exactly. Adjust the window until you can see the vertex and relevant intercepts. An off-screen point is not a missing point. Use a table or substitution to confirm coordinates.

I can...

- ☐ Distinguish a vertex point from its minimum or maximum value.
- ☐ Read intercepts, zeros and the axis of symmetry.
- ☐ Keep a context domain separate from a graph window.

Next class: transform the parent parabola. Curriculum focus: A.6A, A.7A.

LESSON 31 | MEET THE IDEA

Transformations of the quadratic parent function

Tuesday, February 16, 2027 . 6:00–7:00 PM Central

By the end, I can...

Predict how a change inside or outside $f(x)$ transforms the parent parabola, and confirm the prediction using points.

For this lesson, $f(x) = x^2$. A transformation changes its graph. Read **where** the number appears before deciding what it does.

New rule	What changes	Example
$af(x)$	Outputs multiply by a .	$2f(x) = 2x^2$
$f(x) + d$	Outputs increase by d .	$f(x) + 2 = x^2 + 2$
$f(x - c)$	Inputs are shifted; the graph moves right by c when $c > 0$.	$f(x - 2) = (x - 2)^2$
$f(bx)$, $b \neq 0$	Horizontal distances scale by $1/ b $; a negative b also reflects across the y -axis.	$f(2x) = (2x)^2 = 4x^2$

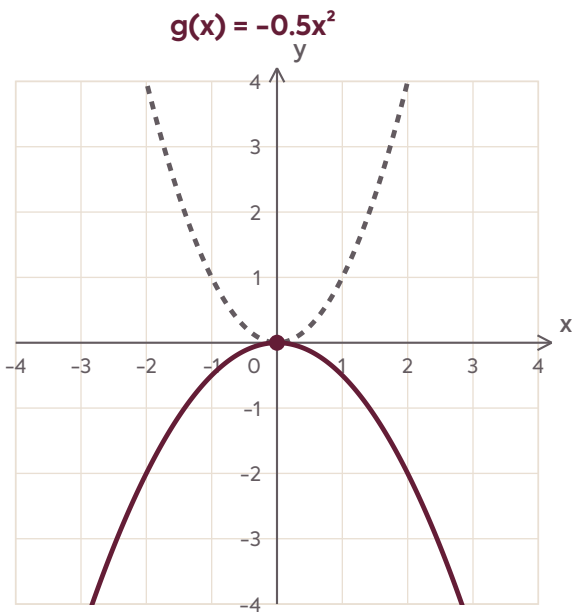
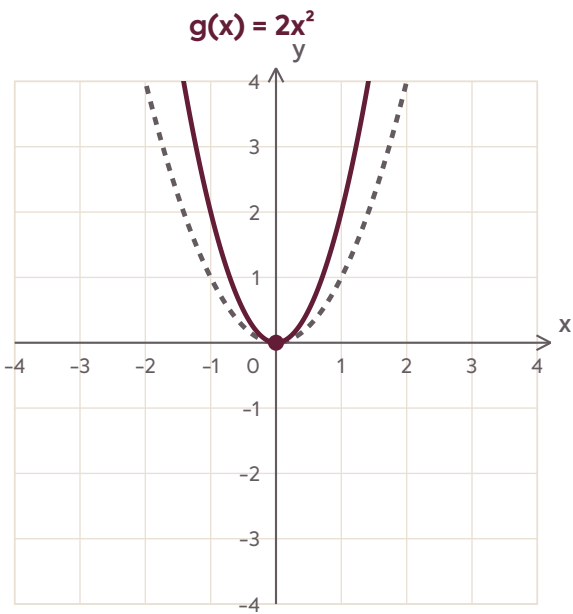
A point is a reliable check

Since $f(1) = 1$, the parent contains $(1, 1)$.
For $g(x) = 2f(x)$, that point becomes $(1, 2)$.
For $g(x) = f(2x)$, the corresponding point becomes $(1/2, 1)$.
The same number 2 appears in different places and does different jobs.

LESSON 31 | VERTICAL SCALING AND REFLECTION

Multiply outside: change the outputs

For $g(x) = af(x)$, a parent point (x, y) becomes **(x, ay)** . The x -coordinate stays the same.



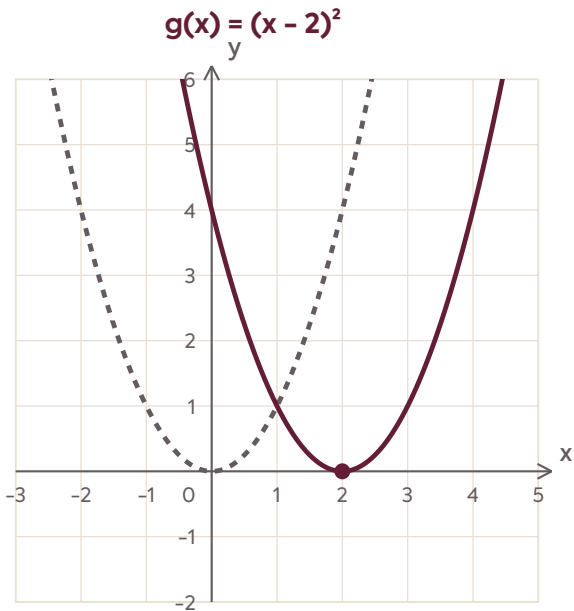
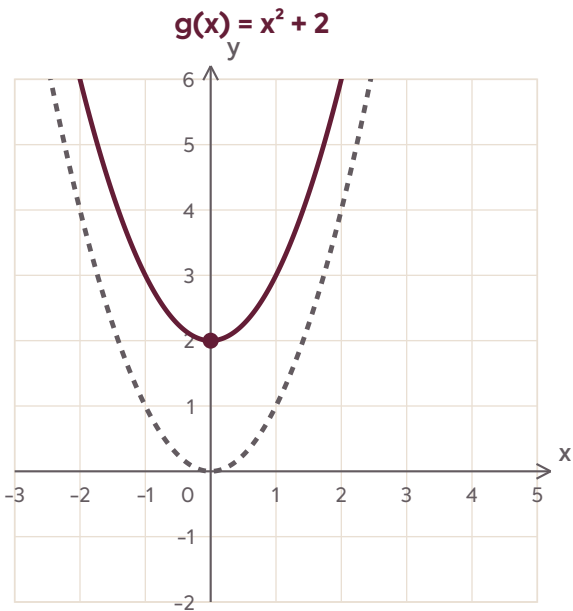
Dashed gray: $y = x^2$. **Solid burgundy:** the transformed graph. The same scale is used on both axes and in both panels.

Value of a	Effect	Point check
$a = 2$	Vertical stretch by 2; opens up.	$(1, 1)$ becomes $(1, 2)$.
$a = -1/2$	Vertical compression by $1/2$ and reflection across the x -axis; opens down.	$(2, 4)$ becomes $(2, -2)$.

For $|a| > 1$, the parabola looks narrower on the same axes. For $0 < |a| < 1$, it looks wider. The sign of a controls the opening. If $a = 0$, the result is constant, not quadratic.

LESSON 31 | VERTICAL AND HORIZONTAL SHIFTS

Add outside; subtract inside



Dashed gray: the parent. Solid burgundy: the transformed graph.

Rule	Movement	Vertex
$f(x) + 2 = x^2 + 2$	Up 2	(0, 2)
$f(x - 2) = (x - 2)^2$	Right 2	(2, 0)
$f(x) - 1.5 = x^2 - 1.5$	Down 1.5	(0, -1.5)
$f(x + 0.5) = (x + 0.5)^2$	Left 0.5	(-0.5, 0)

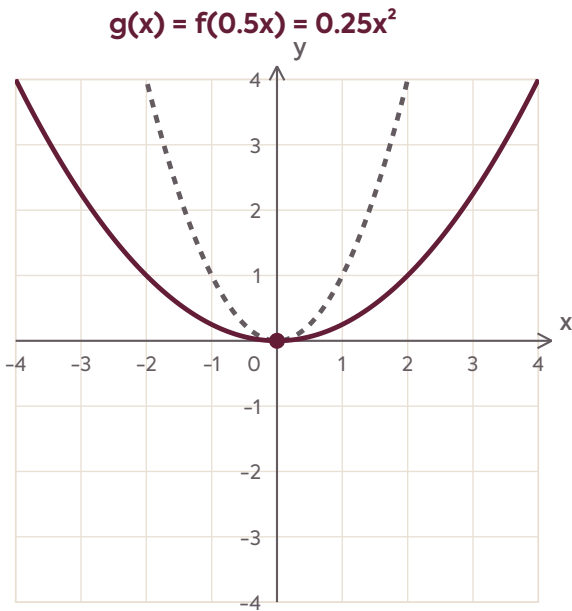
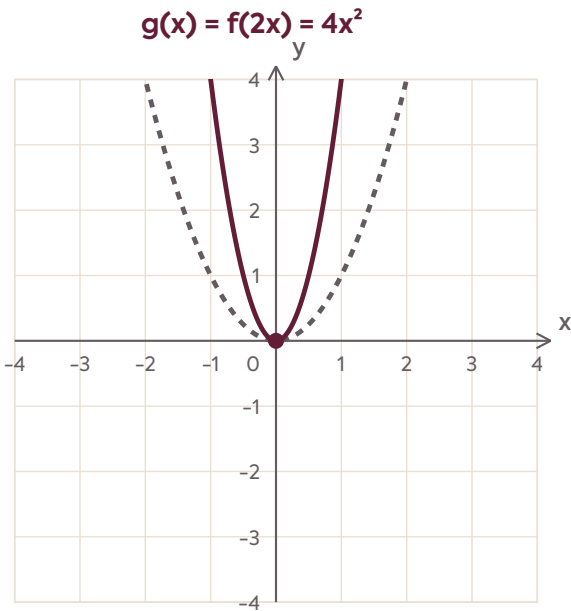
Why $x - 2$ shifts right

The parent has its lowest output when its input is 0. For $(x - 2)^2$, the inside becomes 0 at $x = 2$. That puts the vertex at (2, 0).

LESSON 31 | HORIZONTAL SCALING

Multiply inside: change the inputs

For $g(x) = f(bx)$, $b \neq 0$, a parent point (x, y) becomes $(x/b, y)$. The new input must make bx equal the old input.



Dashed gray: the parent. **Solid burgundy:** the transformed graph.

b	Horizontal change	Check with parent point (2, 4)
2	Compress distances by 1/2.	(1, 4) lies on $f(2x)$.
1/2	Stretch distances by 2.	(4, 4) lies on $f(x/2)$.

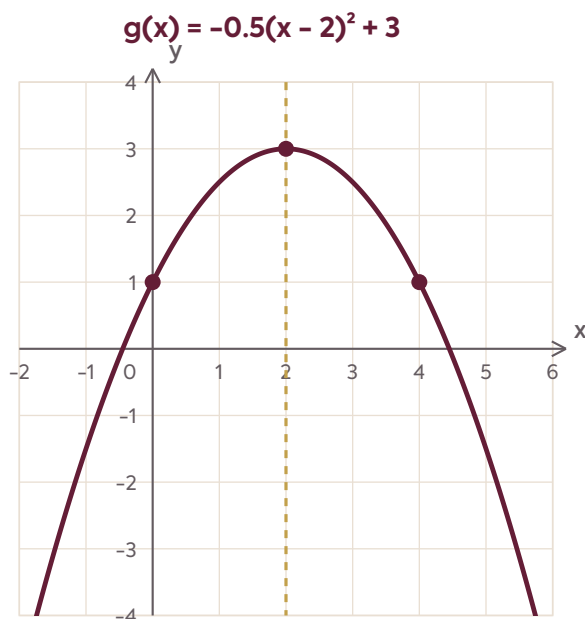
Negative input scaling does not open this parabola down

$f(-2x) = (-2x)^2 = 4x^2 = f(2x)$.
Reflecting $y = x^2$ across the y-axis leaves the same shape because the parent is symmetric. Also, $f(-x/2) = x^2/4$. A negative multiplier **outside** f would open it down.

LESSON 31 | READ VERTEX FORM

Put the changes together

In $g(x) = a(x - h)^2 + k$, the vertex is (h, k) . A parent point (u, u^2) becomes $(u + h, au^2 + k)$. This point rule keeps combined changes clear.



Vertex: $(2, 3)$

Axis: $x = 2$

Opening: downward

Maximum: 3

Range: $y \leq 3$

The points $(0, 1)$ and $(4, 1)$ are equally far from the axis.

Use the point rule for $g(x) = -0.5(x - 2)^2 + 3$

$(0, 0)$ becomes $(2, 3)$

Add 2 to the input; multiply the output by -0.5 , then add 3.

$(2, 4)$ becomes $(4, 1)$

$2 + 2 = 4$ and $-0.5(4) + 3 = 1$.

$(-2, 4)$ becomes $(0, 1)$

The equal outputs confirm symmetry around $x = 2$.

Here the parent is vertically compressed, reflected across the x -axis, shifted right 2 and shifted up 3. All real x are allowed. The maximum value is 3.

LESSON 31 | SHORT CHECKS

Your turn

Throughout, $f(x) = x^2$. Explain each answer using the formula or a point, not just how the curve looks.

1. For $g(x) = -3f(x)$, describe the changes and map the parent point (1, 1).

2. For $g(x) = f(x) - 2.5$, give the vertex and range.

3. For $g(x) = f(x + 3)$, give the horizontal shift and vertex.

4. Simplify $f(3x)$. Describe its horizontal scaling and map the parent point (3, 9).

5. Simplify $f(-x/2)$. Does it open up or down? Map (2, 4) using the input rule.

6. For $g(x) = 2(x + 1)^2 - 4$, give the vertex, axis, opening and range.

7. Are $2f(x)$ and $f(2x)$ the same function? Show a value that decides.

LESSON 31 | WORKED ANSWERS

Check inside versus outside

1. **Vertical stretch by 3 and reflection across the x-axis.** It opens down. The point (1, 1) becomes **(1, -3)**.
2. It shifts down 2.5. Vertex: **(0, -2.5)**; range: **$y \geq -2.5$** .
3. **Left 3**, with vertex **(-3, 0)**. The inside $x + 3$ equals 0 when $x = -3$.
4. $f(3x) = (3x)^2 = 9x^2$. Horizontal distances compress by **$1/3$** . The point (3, 9) becomes **(1, 9)**.
5. $f(-x/2) = (-x/2)^2 = x^2/4$, so it opens **up**. With $b = -1/2$, (2, 4) maps to $(2 \div (-1/2), 4) = \mathbf{(-4, 4)}$. The graph is wider than the parent.
6. Write $x + 1$ as $x - (-1)$. Vertex: **(-1, -4)**; axis: **$x = -1$** ; opening: **upward**; range: **$y \geq -4$** .
7. **No**. $2f(x) = 2x^2$, while $f(2x) = 4x^2$. At $x = 1$, the outputs are **2** and **4**.

I can...

- ☐ Locate a parameter inside or outside the function.
- ☐ Confirm a transformation by mapping a point.
- ☐ Distinguish a negative input scale from an output reflection.

Next class: build a quadratic from a vertex and a point. Curriculum focus: A.7C.

LESSON 32 | MEET THE IDEA

Writing quadratics from a vertex and point; Unit 5 checkpoint

Thursday, February 18, 2027 . 6:00–7:00 PM Central

By the end, I can...

Write a quadratic from its vertex and a second point, expand it into standard form, and use the chapter checkpoint to identify one next step.

Start with $y = a(x - h)^2 + k$. The vertex gives h and k . A second point with $x \neq h$ lets you find a . For a quadratic, a must be nonzero.

Vertex (1, -3), through (3, 5)

$$y = a(x - 1)^2 - 3$$

Put the vertex coordinates into h and k .

$$5 = a(3 - 1)^2 - 3$$

The point gives $x = 3$ and $y = 5$. Substitute both.

$$8 = 4a, \text{ so } a = 2$$

Add 3 to both sides, then divide by 4.

$$y = 2(x - 1)^2 - 3$$

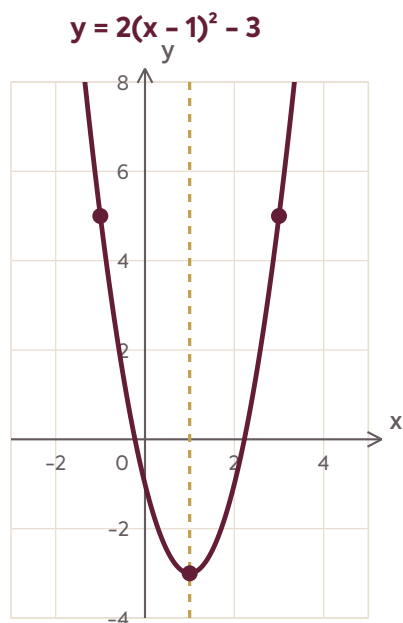
Write the whole function, not just the value of a .

A vertex alone does not fix the width

$y = (x - 1)^2 - 3$ and $y = 2(x - 1)^2 - 3$ have the same vertex but different shapes. The second point decides which curve fits.

LESSON 32 | VERTEX FORM TO STANDARD FORM

Check the graph; change the form



Given vertex: (1, -3)

Given point: (3, 5)

Symmetric point: (-1, 5)

It is also 2 units from $x = 1$.

The graph and substitution both check the equation.

Expand $y = 2(x - 1)^2 - 3$

$$y = 2(x^2 - 2x + 1) - 3$$

$$y = 2x^2 - 4x + 2 - 3$$

$$y = 2x^2 - 4x - 1$$

Square the binomial, including the middle term.

Distribute 2 to every term in the parentheses.

Combine the constants. This is standard form.

Point check: at $x = 3$, standard form gives $18 - 12 - 1 = 5$.

Vertex check: at $x = 1$, it gives $2 - 4 - 1 = -3$.

The two forms describe the same function; one highlights the vertex and the other shows the polynomial coefficients.

LESSON 32 | A SECOND WORKED EXAMPLE

A negative coefficient can fit too

Vertex $(-2, 4)$, through $(0, 2)$

$$y = a(x + 2)^2 + 4$$

$$2 = a(0 + 2)^2 + 4$$

$$-2 = 4a, \text{ so } a = -1/2$$

$$y = -(1/2)(x + 2)^2 + 4$$

Since $h = -2$, $x - h$ becomes $x + 2$.

Substitute the point $(0, 2)$.

Subtract 4, then divide by 4.

The negative a means the graph opens downward.

Rewrite the same function in standard form

$$y = -(1/2)(x^2 + 4x + 4) + 4$$

Expand the squared binomial.

$$y = -(1/2)x^2 - 2x + 2$$

Multiply every term by $-1/2$, then combine constants.

Check the point: $y(0) = 2$. Check the vertex: $y(-2) = -2 + 4 + 2 = 4$. The range is $y \leq 4$.

When the information is not enough

A second point with $x = h$ does not determine a : it is either the same vertex or conflicts with it. If a second point has $x \neq h$ but $y = k$, the calculation gives $a = 0$, so no nonconstant quadratic has that vertex and point.

LESSON 32 | REVIEW AND LESSON TIMING

Connect the chapter before the checkpoint

What you are given	What to do	What to check
A polynomial to factor	GCF first; choose a pattern or grouping.	Multiply the factors back out.
A quadratic graph	Read its vertex, axis, intercepts and range.	Distinguish an x-value from a coordinate pair.
A transformed parent	Identify changes inside and outside $f(x)$.	Map a parent point.
A vertex and point	Insert h and k ; substitute the point; solve for a .	Check both points and expand carefully.

Our one-hour lesson

- 5 minutes: retrieval from earlier lessons.
- 15 minutes: writing a quadratic from a vertex and point.
- 20 minutes: guided examples and checking forms.
- 15 minutes: the Unit 5 checkpoint.
- 5 minutes: exit reflection and next steps.

The checkpoint is supplied separately by your teacher. It samples earlier factoring and graph skills as well as today's work. These notes contain revision questions and their answers, not the graded checkpoint or its answer key.

After the checkpoint

Choose one error to repair. Write the rule you needed, revisit its lesson, and try a fresh question. Use the result to choose your next practice task.

LESSON 32 | STUDY CHECKS, NOT THE GRADED CHECKPOINT

Your turn

For Questions 1-2, write vertex form and standard form. Verify the given vertex and point by substitution.

1. Vertex (2, -1), through (4, 7).

2. Vertex (-1, 3), through (1, 1).

3. Factor $3x^2 - 27$ completely.

4. For $y = -(x - 2)^2 + 5$, give the vertex, axis, maximum value and range.

5. If $f(x) = x^2$, compare $-f(x)$ with $f(-x)$. Which one opens down?

6. Why does giving the vertex (1, 2) and the same point (1, 2) fail to determine one quadratic?

LESSON 32 | WORKED ANSWERS

Check your model and choose a next step

1. $y = a(x - 2)^2 - 1$. Substitute (4, 7): $7 = 4a - 1$, so $a = 2$.

$$y = 2(x - 2)^2 - 1 = 2x^2 - 8x + 7.$$

At $x = 2$: $8 - 16 + 7 = -1$. At $x = 4$: $32 - 32 + 7 = 7$.

2. $y = a(x + 1)^2 + 3$. Substitute (1, 1): $1 = 4a + 3$, so $a = -1/2$.

$$y = -(1/2)(x + 1)^2 + 3 = -(1/2)x^2 - x + 5/2.$$

At $x = -1$: $-1/2 + 1 + 5/2 = 3$. At $x = 1$: $-1/2 - 1 + 5/2 = 1$.

3. $3(x^2 - 9) = 3(x - 3)(x + 3)$. GCF first, then a difference of squares.

4. Vertex: (2, 5); axis: $x = 2$; maximum value: 5; range: $y \leq 5$ for the unrestricted function.

5. $-f(x) = -x^2$ opens down. By contrast, $f(-x) = (-x)^2 = x^2$, which opens up.

6. Substitution gives $2 = a(1 - 1)^2 + 2$, or $2 = 2$, for every a . It gives no new information. Any nonzero a in $y = a(x - 1)^2 + 2$ gives a quadratic with that vertex.

My next learning step

I will revisit Class _____ because _____.

I will check my new attempt by _____.

Chapter 6 begins Tuesday, February 23, 2027: quadratic equations and modeling. We will connect factors to zeros and learn ways to solve quadratic equations. Curriculum focus: A.6B; review A.10D-F, A.6A and A.7A,C.

KEEP THIS PAGE FOR REVISION

Chapter 5 quick reference

Idea	Remember	Example
GCF first	Take out a factor shared by every term before looking for other patterns.	$2x^2 + 10x + 12 = 2(x + 2)(x + 3)$
Monic trinomial	For $x^2 + bx + c$, find $mn = c$ and $m + n = b$.	$x^2 - x - 6 = (x - 3)(x + 2)$
Nonmonic trinomial	Find product ac and sum b ; split the middle term and group.	$2x^2 + 7x + 3 = (2x + 1)(x + 3)$
Perfect square	Check the middle term, $\pm 2uv$.	$u^2 \pm 2uv + v^2 = (u \pm v)^2$
Difference of squares	Subtraction between two squares. Keep the opposite signs.	$u^2 - v^2 = (u - v)(u + v)$
Vertex form	$y = a(x - h)^2 + k$, $a \neq 0$. Vertex (h, k) ; axis $x = h$.	$a > 0$: opens up, $y \geq k$. $a < 0$: opens down, $y \leq k$. (Full quadratic.)
Inside / outside	$af(x)$ changes outputs; $f(bx)$ changes inputs. For the parent x^2 , $f(bx) = b^2x^2$.	$-f(x)$ opens down; $f(-x) = x^2$ opens up.
Vertex and point	Insert h and k . Substitute a second point with $x \neq h$ to solve for a .	Then expand $(x - h)^2$, distribute a and combine.

A context may restrict the domain and range. An unsuccessful integer-factor search does not rule out real factors. Check every factorization by expansion.

STUDY GUIDANCE AND CURRICULUM REFERENCES

Make these notes work for you

A small routine for each lesson

Before: recall the prerequisite named in the lesson.

During: predict the next worked step and explain why it works.

After: try the study checks with the answers covered, then do the digital practice your teacher assigns.

Study checks and graded assessments

These notes include answers to their own short practice checks. The Class 28 quiz and Class 32 graded checkpoint are separate teacher-provided materials. Their unpublished questions and answer keys are not included.

Read graphs precisely

Each graph labels its axes and scale; curves are drawn from the stated equations. Unrestricted parabolas continue beyond the displayed window. The height model shows only its stated time interval. Dashed gray curves in transformation comparisons are the parent; solid burgundy curves are the transformed function.

Curriculum alignment

Standard Algebra I, §111.39(c): A.6A–B, A.7A,C and A.10D–F. Advanced Grade 8 Algebra I, §111.31(d): (6)(A)–(B), (7)(A),(C) and (10)(D)–(F). Earlier algebra skills and mathematical processes are revisited throughout.

These are original Cambridge Veritas supplementary notes, not an official TEA publication. The lesson dates are our agreed course calendar, not state-required pacing. Contexts and student names in examples are fictional. Chapter 6 develops the full connection between factors, real zeros and solving equations.

Official references

Texas Education Agency: [Current mathematics standards index](#).

Standard Algebra I: [§111.39, high-school mathematics TEKS](#).

Advanced Grade 8 Algebra I: [§111.31, middle-school mathematics TEKS, adopted 2025](#).

Standards checked September 9, 2026. Chapter 5 edition: 1.0.