

LESSON 34 | MEET THE IDEA

Solving quadratics by taking square roots

Thursday, February 25, 2027 . 6:00–7:00 PM Central

By the end, I can...

Isolate a squared expression, find all its real solutions and decide which solutions make sense in a context.

Some quadratics already contain a square you can isolate. The method is: **isolate the square, take square roots, then solve the remaining linear equations.**

Solve $x^2 = 25$

$$x = \pm\sqrt{(25)}$$

Both 5 and -5 square to 25. The $\pm$ sign means plus OR minus.

$$x = 5 \text{ or } x = -5$$

List the two possible values.

A radical symbol and an equation do different jobs

$\sqrt{(25)} = 5$: the radical symbol gives the principal, nonnegative square root.

$x^2 = 25$: the equation has two solutions, ± 5 .

Do not write $\sqrt{(25)} = \pm 5$.

Solve $3x^2 = 48$

$$x^2 = 16$$

Divide both sides by 3 before taking square roots.

$$x = \pm 4$$

Check: $3(4^2) = 48$ and $3((-4)^2) = 48$.

LESSON 34 | SHIFTED SQUARES AND EXACT ANSWERS

Isolate the whole squared expression

Solve $2(x - 3)^2 = 24$

$(x - 3)^2 = 12$

$x - 3 = \pm\sqrt{12}$

$x - 3 = \pm 2\sqrt{3}$

$x = 3 \pm 2\sqrt{3}$

Exact check: for either root, $x - 3 = \pm 2\sqrt{3}$, so $2(x - 3)^2 = 2 \times 12 = 24$.

Divide both sides by 2.

Take both square roots of 12.

$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$.

Add 3 to both possibilities.

Exact solution	Approximate solution
$3 + 2\sqrt{3}$	6.46, to two decimal places
$3 - 2\sqrt{3}$	-0.46, to two decimal places

Keep exact values until the last step

$3 \pm 2\sqrt{3}$ is exact. Decimal values are rounded, so write $\approx$ when using them. Substituting a rounded value may give an answer close to 24 rather than exactly 24.

LESSON 34 | REAL SOLUTIONS AND SENSIBLE LENGTHS

Check what the square equals

Isolated square	Real solutions	Reason
$(x - h)^2 = k, k > 0$	$x = h \pm \sqrt{k}$	A positive number has two opposite real square roots.
$(x - h)^2 = 0$	$x = h$	Only zero squares to zero.
$(x - h)^2 = k, k < 0$	No real solution	A real number squared is never negative.

A square has area 75 cm^2 . Find its side length s .

$s^2 = 75$
 $s = \pm\sqrt{75} = \pm 5\sqrt{3}$
 $s = 5\sqrt{3} \text{ cm} \approx 8.66 \text{ cm}$

Area of a square equals side length squared.

First find both algebraic possibilities.

A side length must be positive, so use the positive value.

The equation decides the candidates; the context selects

A negative solution can be mathematically correct but unsuitable for a length. Do not reject a negative value just because it is negative: temperatures, positions and other quantities may legitimately be below zero.

LESSON 34 | SHORT CHECKS

Your turn

1. Solve $x^2 = 49$.

2. Solve $2(x + 1)^2 = 18$.

3. Solve $3(x - 2)^2 = 30$. Give exact solutions and approximations to two decimal places.

4. Solve $(x + 4)^2 = 0$.

5. A square has area 50 m^2 . Give its side length exactly and to two decimal places. Explain your root selection.

6. Does $4(x - 1)^2 + 7 = 3$ have real solutions? Explain after isolating the square.

LESSON 34 | WORKED ANSWERS

Keep both signs when they are needed

1. $x = \pm 7$. Both 7^2 and $(-7)^2$ equal 49.

2. $(x + 1)^2 = 9$, so $x + 1 = \pm 3$. Thus $x = 2$ or $x = -4$. Each makes $2(x + 1)^2$ equal 18.

3. $(x - 2)^2 = 10$, so $x = 2 \pm \sqrt{10}$.

The approximate solutions are **5.16 and -1.16**. Exact check: $3(\pm\sqrt{10})^2 = 30$.

4. $x + 4 = 0$, so $x = -4$. There is only one distinct real solution.

5. $s^2 = 50$ gives $s = \pm 5\sqrt{2}$. A side length is positive, so $s = 5\sqrt{2} \text{ m} \approx 7.07 \text{ m}$. The negative algebraic root is unsuitable for a length.

6. $4(x - 1)^2 = -4$, so $(x - 1)^2 = -1$. There is **no real solution** because a real square cannot be negative.

A reliable checking routine

Put each exact candidate back into the **original** equation. Then check the units and restrictions of any story. Do not round early or forget the second sign when the isolated square is positive.

I can...

- ☐ Isolate a square before taking square roots.
- ☐ Distinguish two, one and no real solution cases.
- ☐ Keep exact radicals and justify contextual root choices.

Next class: create a square when one is not already visible. Curriculum focus: A.8A.