

LESSON 44 | MEET THE IDEA

Choosing linear, quadratic or exponential models

Thursday, April 8, 2027 . 6:00–7:00 PM Central

By the end, I can...

Choose a model family using tables, graphs and context, then explain what a fitted model can and cannot tell us.

You already know three important families. Today, choose between them by asking **how the output changes when the input changes by equal amounts**. An increasing graph alone is not enough.

Family	Form	Useful clue
Linear	$y = mx + b$	A fixed amount is added for each one-unit input step.
Quadratic	$y = ax^2 + bx + c, a \neq 0$	First differences change by a fixed amount.
Exponential	$y = ab^x$	A fixed multiplier applies for each one-unit input step.

Start with the story

A service charges a fixed fee plus a constant amount per hour: consider a **linear** model.

A square’s area changes with its side length: consider a **quadratic** model.

A positive quantity changes by the same percentage each period: consider an **exponential** model.

Use $b > 0$, $b \neq 1$ and $a \neq 0$ for a standard nonconstant real exponential model. The letter b has a different role in each family; interpret it from the formula.

LESSON 44 | EQUAL INPUT STEPS MATTER

Read the pattern in an exact table

x	0	1	2	3	4
L(x)	3	7	11	15	19
Q(x)	1	2	5	10	17
E(x)	3	6	12	24	48

Compare neighboring values

L: 4, 4, 4, 4

First differences are constant, so $L(x) = 3 + 4x$ fits.

Q: 1, 3, 5, 7

First differences are not constant. Their differences are 2, 2, 2.

$Q(x) = x^2 + 1$

A quadratic matches this constant nonzero second difference.

E: 2, 2, 2, 2

Consecutive ratios are constant, so $E(x) = 3 \times 2^x$ fits.

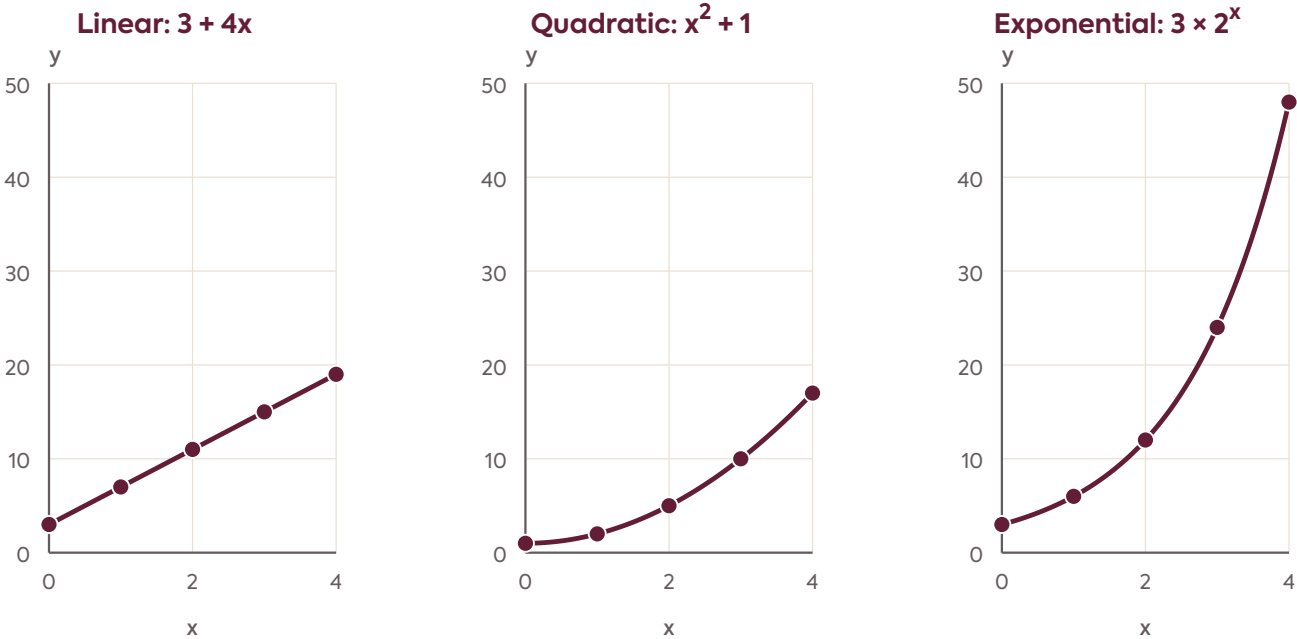
Use these tests with their conditions

Inputs must be **equally spaced**. Ratio tests also need nonzero previous outputs. A constant first difference suggests a line; a constant **nonzero** second difference suggests a quadratic. For step size h , a quadratic's second difference is $2ah^2$.

Here we assume an exact pattern from one of these families. A finite table does not prove what all later values must be, and rounded measurements will rarely follow an exact pattern.

LESSON 44 | LOOK BEYOND “IT GOES UP”

Compare graphs using the same scales



All three panels use x from 0 to 4 and y from 0 to 50. The dots are the five table entries. Each formula also gives outputs between them, so these illustrations show continuous curves on that interval.

What you notice	What it means
The linear graph is straight.	Its slope stays constant: equal horizontal steps give equal vertical changes.
The quadratic graph curves.	Its slope changes; this quadratic has a turning point at (0, 1).
The exponential graph rises faster here.	Its positive output doubles each step. The amount added also gets larger.

Match the domain to the quantity

Use a continuous curve for a continuous quantity. For counts or sequences, include only allowed integer inputs.

LESSON 44 | COMPARE MODELS FOR NOISY DATA

A tiny fitting error is not the whole decision

Return to the fictional signal data from Class 43: times 0, 1, 2, 3, 4, 5 hours and readings 100, 81, 63, 52, 40, 33 units. Three original-output least-squares fits are shown below.

Family and fitted rule	Total squared error
Linear: $L(t) = 95 - 13.4t$	87.20
Quadratic: $Q(t) = 1.5t^2 - 20.9t + 100$	3.20
Exponential: $E(t) \approx 100.291(0.798844)^t$	3.48

Squared error adds $(\text{observed} - \text{predicted})^2$ across the six readings. For this comparison it has squared signal units. The quadratic has the smallest error **on these data**, but the future shape still needs a reason.

Look at the assumptions behind a prediction

$L(10) = -39$

A negative signal is unsuitable for a nonnegative intensity model.

$Q(10) = 41$

The quadratic turns upward after about 6.97 hours.

$E(10) \approx 10.61$

The exponential continues fading toward zero.

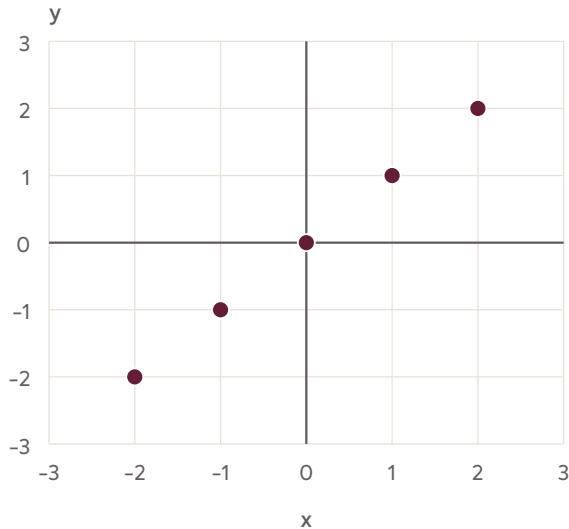
If the process is expected to keep losing roughly the same percentage, the exponential is a reasonable candidate. We need new observations to test that assumption.

For a guided check, refit the table using the linear and quadratic regression forms from earlier lessons. Keep Log Mode off for the exponential comparison. Compare the curves on 0–5 hours before extending them. See the [official regression guide](#).

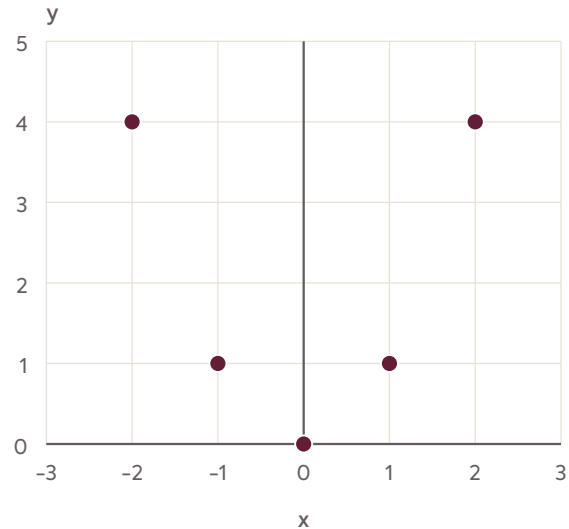
LESSON 44 | WHAT CORRELATION ACTUALLY MEASURES

Association is not a cause

A straight-line association



A perfect curved pattern



Use a graphing calculator's linear-regression analysis on each paired table. For both, $x = -2, -1, 0, 1, 2$. On the left, $y = -2, -1, 0, 1, 2$; on the right, $y = 4, 1, 0, 1, 4$. The linear correlation coefficients are $r = 1$ and $r = 0$, respectively.

r measures linear association

r is between -1 and 1 . Its sign gives the direction of a linear trend; a value near -1 or 1 indicates a strong linear association.

The right-hand dots fit $y = x^2$ perfectly, although $r = 0$. A small r does not rule out a strong **nonlinear** relationship.

Suppose ice-cream sales and pool visits rise together. That association does not prove that buying ice cream causes pool visits. Hot weather could help explain both. Observing a pattern alone does not establish causation.

10-minute quiz . 15-minute explanation . 20-minute guided comparison . 10-minute independent practice . 5-minute exit check. The teacher supplies the quiz separately; the study checks are revision materials.

LESSON 44 | SHORT CHECKS

Your turn

For Questions 1–2, $x = 0, 1, 2, 3$ and the listed patterns are assumed exact within one of the three families.

1. Classify each output list and write a matching rule: (a) 5, 8, 11, 14; (b) 2, 5, 10, 17.

2. Classify 6, 18, 54, 162. Give a rule and predict the output at $x = 4$.

3. For the signal data, which fitted family has the smallest squared error? Why does that alone not settle the best model for continued fading?

4. A scatterplot follows a clear U-shape but has r close to zero. Does that mean there is no association? Explain.

5. A school finds that students who join a club get higher grades. Explain why this does not prove the club caused the higher grades.

6. Why is a signal prediction at $t = 10$ less secure than one at $t = 2.5$ when observations cover only 0–5 hours?

LESSON 44 | WORKED ANSWERS

Choose a model and give a reason

1. (a) Differences are 3, 3, 3: **linear**, $y = 5 + 3x$.

(b) First differences are 3, 5, 7, so second differences are 2, 2: **quadratic**, $y = x^2 + 2x + 2$. Substituting $x = 0, 1, 2, 3$ gives the stated outputs.

2. Ratios are 3, 3, 3: **exponential**, $y = 6 \times 3^x$. At $x = 4$, $y = 6 \times 81 = 486$.

3. The **quadratic** has the smallest error, 3.20 versus about 3.48 for the exponential. But its upward turn would contradict an assumption of continued fading. Context, model behavior and further data also matter.

4. **No**. r measures the strength of a **linear** association. A U-shaped pattern can have strong nonlinear association with r near zero.

5. Students may differ in motivation, prior grades or available support before joining. These or other factors could explain the association. The observation alone does not establish a causal effect.

6. $t = 10$ is **extrapolation** beyond the observed interval; the conditions or pattern may change. $t = 2.5$ is **interpolation** inside the observed interval. Neither is a guaranteed observation.

I can...

- ☐ Use equal-step differences and ratios appropriately.
- ☐ Compare fitted models using both data and context.
- ☐ Explain what correlation and extrapolation cannot prove.

Next class: review the specific skills your work shows you still need. Curriculum focus: A.4A–C, A.8B, A.9E; earlier graph and model skills.