

LESSON 15 | MEET THE IDEA

Two-variable linear inequalities

Tuesday, December 8, 2026 . 6:00–7:00 PM Central

By the end, I can...

Write a linear inequality in two variables, graph its solution region and explain whether a point belongs.

An equation such as $x + y = 4$ describes a line. An inequality such as $x + y \leq 4$ also allows points on one side of that line. Its solutions are **ordered pairs** (x, y) .

Point	Substitute in $x + y \leq 4$	Solution?
(1, 2)	$1 + 2 = 3$, and $3 \leq 4$.	Yes
(1, 3)	$1 + 3 = 4$, and $4 \leq 4$.	Yes: on the boundary
(3, 2)	$3 + 2 = 5$, and $5 \leq 4$ is false.	No

Four steps for the graph

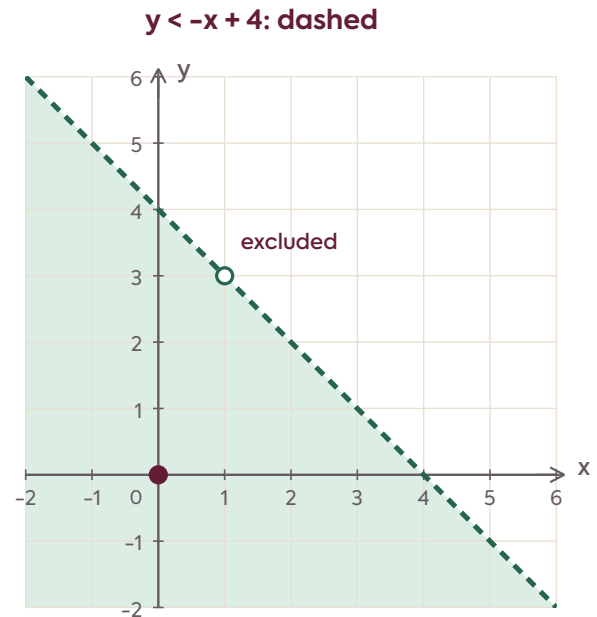
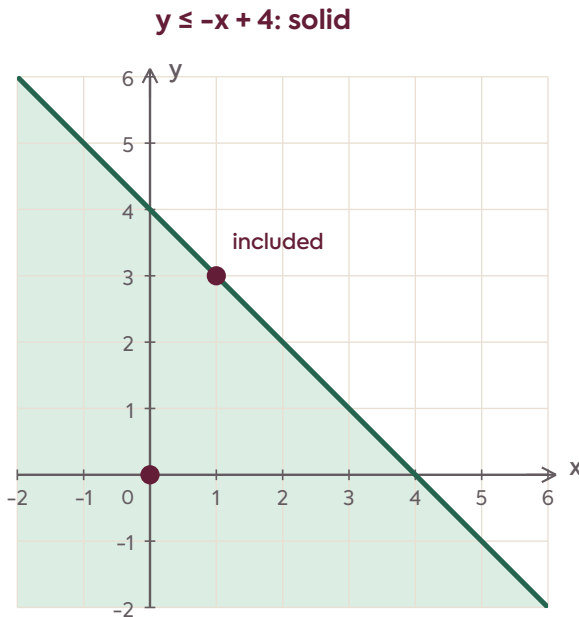
1. Replace the inequality sign with $=$ to draw the boundary.
2. Use solid for $\leq$ or $\geq$; dashed for $<$ or $>$.
3. Test a point that is not on the boundary.
4. Shade the side that makes the original inequality true.

A straight boundary divides the plane into two half-planes. Points on the same side give the same true/false result, which is why one test point can choose the side.

LESSON 15 | SEE THE DIFFERENCE

Does the boundary count?

Both graphs shade below $y = -x + 4$. Only the left graph includes the boundary line. The shaded regions continue beyond the grids shown.



Test $(0, 0)$: $0 < 4$ and $0 \leq 4$ are both true, so shade the side containing the origin. A point on the boundary cannot choose the side.

Be careful when you divide by a negative number

$$x - 2y < 4$$

Subtract x from both sides.

$$-2y < 4 - x$$

Divide both sides by -2 and reverse the sign.

$$y > (1/2)x - 2$$

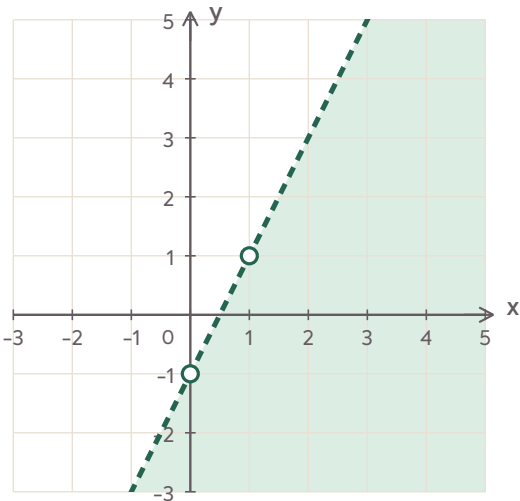
Use a dashed boundary and shade above it.

LESSON 15 | GRAPHS AND TABLES

Write the rule from evidence

Read this graph: the dashed boundary passes through (0, -1) and (1, 1). The shaded side lies below it. What inequality does it show?

Read the boundary and shaded side



Slope = $(1 - (-1)) / (1 - 0) = 2$; y-intercept = -1. Dashed means equality is excluded. Shading below gives $y < 2x - 1$.

For boundary $x + y = 4$	Inside shaded region?
(0, 0)	Yes
(1, 3)	Yes, including the boundary
(3, 2)	No

Choose from $x + y < 4$, $x + y \leq 4$, $x + y > 4$, $x + y \geq 4$. The table selects $x + y \leq 4$. Without the supplied boundary and choices, a few points would not determine a unique inequality.

LESSON 15 | A REAL-WORLD MODEL

Turn a budget into a region

A notebook costs \$3 and a pen costs \$2. You can spend **at most \$12**. Let x be notebooks and y be pens. The budget rule is $3x + 2y \leq 12$, with x and y nonnegative whole numbers.

Every dot is an affordable whole-number choice



The solid boundary shows spending exactly \$12, through (0, 6) and (4, 0). Shading below shows spending no more than \$12. **Only the dots** represent allowed purchases here; you cannot buy a fraction of a notebook.

Choice	Cost	Allowed?
(2, 3)	$\$6 + \$6 = \$12$	Yes
(3, 2)	$\$9 + \$4 = \$13$	No

Translate carefully
At most / no more than: $\leq$. At least / no less than: $\geq$.
Fewer than / less than: $<$. More than: $>$. Define your variables and units before choosing a symbol.

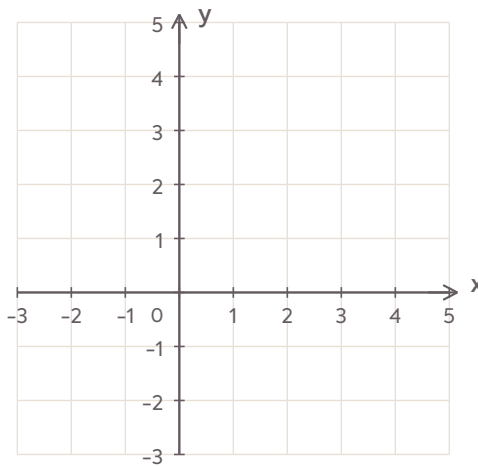
LESSON 15 | SHORT CHECKS

Your turn

Try these before reading the worked answers. Your main homework is in digital practice.

1. Graph $y > x - 1$ on the grid. Mark the boundary correctly and test $(0, 0)$.

Your solution region



2. Which points satisfy $x + 2y \leq 6$: $(2, 2)$, $(4, 2)$, $(0, 3)$? Show the substitutions.

3. Rearrange $x - 2y \leq 4$ to make y the subject. Is its region above or below the boundary?

4. Small kits cost \$4 and large kits cost \$7. Write the rule for spending at most \$28. Include sensible restrictions on the numbers of kits.

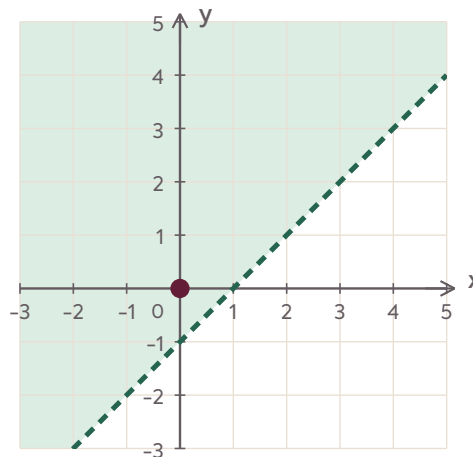
5. A solid boundary passes through $(0, 2)$ and $(2, 0)$; the shaded side includes $(0, 0)$. Write the inequality.

LESSON 15 | WORKED ANSWERS

Check the side, sign and meaning

1. Draw $y = x - 1$ **dashed** and shade above it. At $(0, 0)$, $0 > -1$ is true. The origin belongs; boundary points do not.

Answer 1: $y > x - 1$



2. **(2, 2): yes**, because $2 + 2(2) = 6 \leq 6$. **(4, 2): no**, because $8 > 6$. **(0, 3): yes**, because $6 \leq 6$. The two solutions lie on the included boundary.

3. $-2y \leq 4 - x$ becomes $y \geq (1/2)x - 2$. Reverse the sign when dividing by -2 ; use a solid boundary and shade above.

4. Let s and l count small and large kits. $4s + 7l \leq 28$, where s and l are nonnegative whole numbers.

5. The boundary is $y = -x + 2$. Solid and below gives $y \leq -x + 2$, or $x + y \leq 2$.

I can...

- ☐ Choose a boundary and shade the correct side.
- ☐ Test a point in the original inequality.

Next class: two equations that must be true together. Curriculum focus: A.2H, A.3D.