

LESSON 39 | MEET THE IDEA

Arithmetic sequences: recursive and explicit rules

Tuesday, March 23, 2027 . 6:00–7:00 PM Central

By the end, I can...

Recognize an arithmetic sequence, write both kinds of rule and connect its discrete graph to a linear relationship.

A **sequence** is an ordered list. Each number is a **term**. The term number tells you a position, not a value. In this lesson we number terms **$n = 1, 2, 3, \dots$**

Term number n	1	2	3	4
Term value a_n	8	13	18	23

Same amount added each time

$13 - 8 = 5$, $18 - 13 = 5$ and $23 - 18 = 5$.

An **arithmetic sequence** has the same difference between consecutive terms. Here the **common difference** is $d = 5$.

Read a_n as "a sub n": the value of term n . Here $a_1 = 8$, $a_2 = 13$ and $a_4 = 23$. The little number is a position label, not an exponent.

Check more than one jump

A short list suggests a pattern; it does not prove how every later term must behave. Use the stated rule or the assumption that the sequence is arithmetic before extending it.

LESSON 39 | START WITH THE INDEX

Two rules, one sequence

Recursive: use the previous term

For 8, 13, 18, 23, ...:
 $a_1 = 8; a_n = a_{n-1} + 5$, for $n = 2, 3, \dots$
The starting value is essential. "Add 5" alone does not tell you where to begin.

Explicit: go straight to any term

$$a_n = a_1 + (n - 1)d$$
$$a_n = 8 + 5(n - 1)$$
$$a_{10} = 8 + 5(9) = 53$$

Starting at term 1, reaching term n takes $n - 1$ jumps.

Use $a_1 = 8$ and $d = 5$; $n = 1, 2, 3, \dots$

Term 10 is nine jumps after the first term.

Check the first term: $8 + 5(1 - 1) = 8$. An explicit rule uses the index directly; it does not require calculating earlier terms.

Rule type	What must be supplied?	Best use
Recursive	First term and a rule using the previous term	Generate terms in order.
Explicit	Formula in n and the allowed indices	Find a distant term directly.

Avoid the extra jump: $8 + 5n$ gives 13 when $n = 1$, so it is not the rule for this sequence with first index 1.

LESSON 39 | DECREASE AND CONTEXT

A difference can be negative

50, 43, 36, 29, ... is arithmetic

$$d = 43 - 50 = -7$$

Subtract 7 each time; adding -7 means the same thing.

$$a_n = 50 - 7(n - 1)$$

The first term is at $n = 1$.

$$a_6 = 50 - 35 = 15$$

There are five steps from term 1 to term 6.

A theater has 12 seats in row 1 and 4 more in each next row.

$$s_n = 12 + 4(n - 1)$$

n is a whole-number row label: 1, 2, 3, ...

$$s_8 = 12 + 4(7) = 40$$

Row 8 contains 40 seats.

$$s_1 = 12; s_n = s_{n-1} + 4$$

This recursive version applies for $n = 2, 3, \dots$

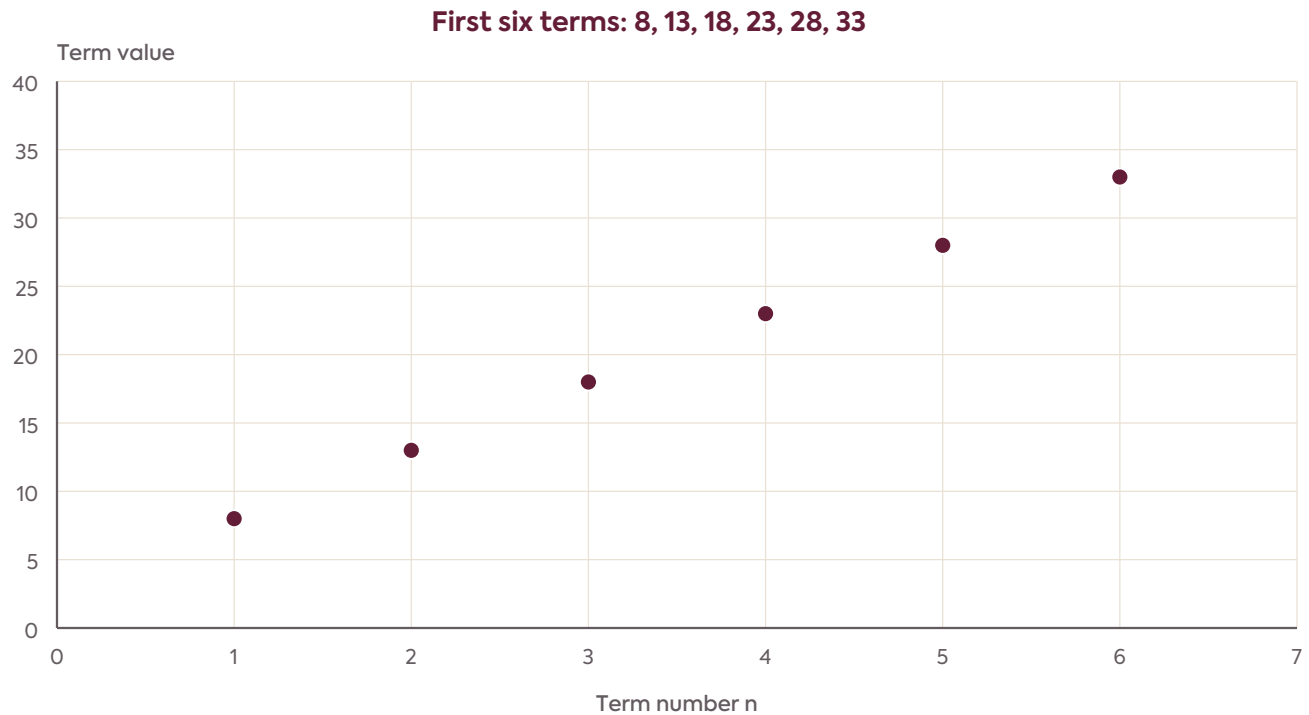
The story limits the rule

If the theater has eight rows, the domain is $n = 1, 2, \dots, 8$. Do not invent row 8.5 or keep extending the physical theater beyond its actual rows.

A zero common difference produces a constant arithmetic sequence, such as 6, 6, 6, Arithmetic means constant difference, not necessarily an increase.

LESSON 39 | CONNECT THE RULE TO A LINE

Dots are the sequence graph



The plotted points are $(1, 8)$, $(2, 13)$, ..., $(6, 33)$. The graph of these six terms contains **only the six dots**. There is no term numbered 2.5, so do not connect the dots as if all inputs were allowed.

The same values lie on a line

$$a_n = 8 + 5(n - 1)$$

Begin with the explicit sequence rule.

$$a_n = 5n + 3$$

Expand and combine constants.

$$y = 5x + 3$$

This continuous line contains the dots, but has a larger domain.

The common difference 5 is the line's rate of change per one index step. This relationship is **not proportional**: the line $y = 5x + 3$ does not pass through the origin. For n starting at 1, an arithmetic rule is proportional to n when $a_1 = d$.

LESSON 39 | SHORT CHECKS

Your turn

Assume every listed sequence continues with the stated arithmetic pattern. Unless stated otherwise, the first term has index $n = 1$.

1. For 3, 7, 11, 15, ... , write recursive and explicit rules. Find term 20.

2. For 18, 15, 12, ... , find d and the index of the first zero term.

3. $a_n = 7 + 2(n - 1)$, $n = 1, 2, \dots$. List the first three terms and find a_{12} .

4. $a_1 = -2$; $a_n = a_{n-1} + 5$ for $n = 2, 3, \dots$. Find terms 2–4 and an explicit rule.

5. Is 7, 14, 21, 28, ... proportional to its term number n ? Give a rule and a reason.

6. Why should the first six terms of a sequence be drawn as six dots? Is $n = 2.5$ an allowed index here?

LESSON 39 | WORKED ANSWERS

Check the start and every jump

1. $d = 4$. Recursive: $a_1 = 3$; $a_n = a_{n-1} + 4$ for $n = 2, 3, \dots$. Explicit: $a_n = 3 + 4(n - 1)$ for $n = 1, 2, \dots$. Term 20 is $3 + 4(19) = 79$.

2. $d = -3$. Set $18 - 3(n - 1) = 0$. Then $n - 1 = 6$, so $n = 7$. The seventh term is zero; the value zero is not its index.

3. At $n = 1, 2, 3$, the values are **7, 9, 11**. $a_{12} = 7 + 2(11) = 29$.

4. Terms 2–4 are **3, 8, 13**. Explicit: $a_n = -2 + 5(n - 1) = 5n - 7$, $n = 1, 2, \dots$. Check $n = 1$ gives -2 .

5. **Yes**. $a_n = 7 + 7(n - 1) = 7n$, $n = 1, 2, \dots$. The ratio a_n/n is always 7. Its dots lie on $y = 7x$, a line through the origin.

6. Only the indices **1, 2, ..., 6** are included in the six-term graph. **2.5 is not an allowed index**. Connecting the dots would add inputs that are not sequence terms.

I can...

- ☐ Separate an index from its term value.
- ☐ Include an initial value and index range in a recursive rule.
- ☐ Explain why an arithmetic sequence is graphed with dots.

Next class: patterns that multiply instead of add. Curriculum focus: A.12C–D; review A.3B and A.2A.