



Cambridge
Veritas™

CHAPTER 08

Algebra I

Model comparison, cumulative review and final assessment

Classes 44–46 . April 8 – April 15, 2027

Tuesdays and Thursdays . 6:00–7:00 PM Central

CONNECT THE IDEAS. SHOW YOUR REASONING.

Compare

Choose a model using evidence and context.

Review

Return to the skills your work shows you need.

Reflect

Use feedback to decide the next learning step.

Worked examples . Accurate graphs . Mixed revision

Correction worksheets . Chapter reference

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Your route through Chapter 8

Bring the course together: compare models, review priority skills and reflect on the final assessment with your teacher.

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Chapter quick reference: page 21 . Study guidance and sources: page 22

How to use the final lesson

Class 46 begins with an instructions page. Its remaining pages are a worksheet for use after teacher review. The graded assessment and its answer key are not included in this book.

The combined chapter is a completion resource. Before final review, use the individual Class 44 and 45 notes and the instructions page from Class 46. The separate one-page formula sheet is for revision unless your teacher explicitly authorizes it during an assessment.

LESSON 44 | MEET THE IDEA

Choosing linear, quadratic or exponential models

Thursday, April 8, 2027 . 6:00–7:00 PM Central

By the end, I can...

Choose a model family using tables, graphs and context, then explain what a fitted model can and cannot tell us.

You already know three important families. Today, choose between them by asking **how the output changes when the input changes by equal amounts**. An increasing graph alone is not enough.

Family	Form	Useful clue
Linear	$y = mx + b$	A fixed amount is added for each one-unit input step.
Quadratic	$y = ax^2 + bx + c, a \neq 0$	First differences change by a fixed amount.
Exponential	$y = ab^x$	A fixed multiplier applies for each one-unit input step.

Start with the story

A service charges a fixed fee plus a constant amount per hour: consider a **linear** model.
A square’s area changes with its side length: consider a **quadratic** model.
A positive quantity changes by the same percentage each period: consider an **exponential** model.

Use $b > 0$, $b \neq 1$ and $a \neq 0$ for a standard nonconstant real exponential model. The letter b has a different role in each family; interpret it from the formula.

LESSON 44 | EQUAL INPUT STEPS MATTER

Read the pattern in an exact table

x	0	1	2	3	4
L(x)	3	7	11	15	19
Q(x)	1	2	5	10	17
E(x)	3	6	12	24	48

Compare neighboring values

L: 4, 4, 4, 4

First differences are constant, so $L(x) = 3 + 4x$ fits.

Q: 1, 3, 5, 7

First differences are not constant. Their differences are 2, 2, 2.

$Q(x) = x^2 + 1$

A quadratic matches this constant nonzero second difference.

E: 2, 2, 2, 2

Consecutive ratios are constant, so $E(x) = 3 \times 2^x$ fits.

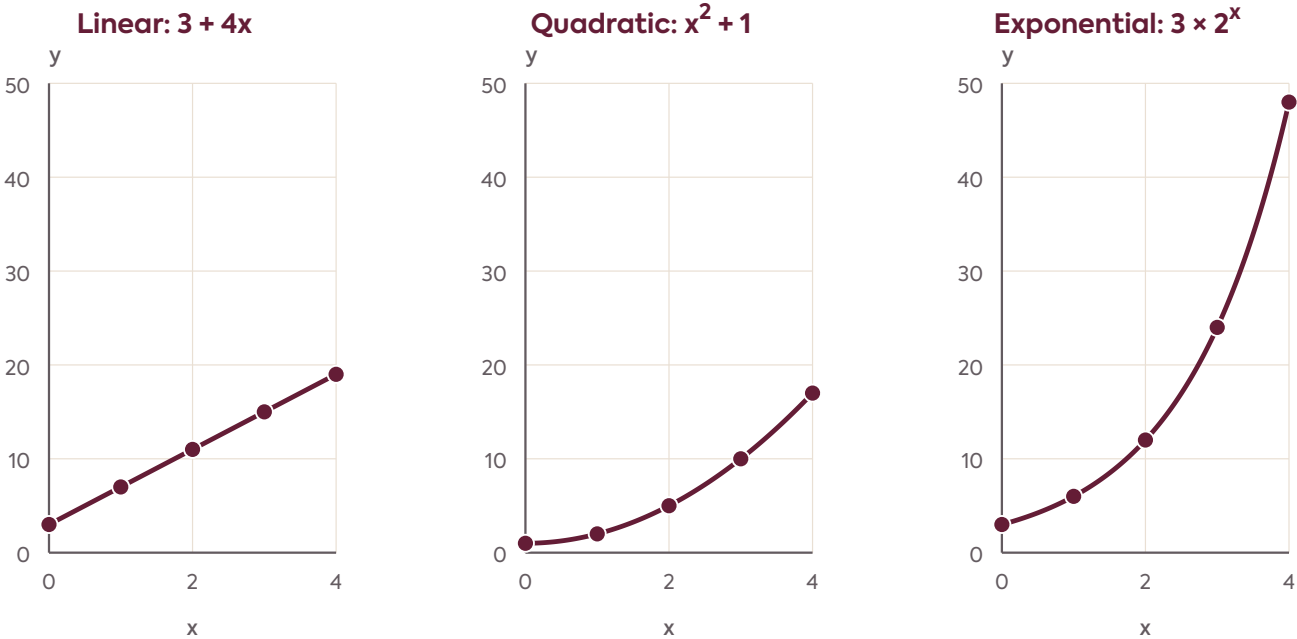
Use these tests with their conditions

Inputs must be **equally spaced**. Ratio tests also need nonzero previous outputs. A constant first difference suggests a line; a constant **nonzero** second difference suggests a quadratic. For step size h , a quadratic's second difference is $2ah^2$.

Here we assume an exact pattern from one of these families. A finite table does not prove what all later values must be, and rounded measurements will rarely follow an exact pattern.

LESSON 44 | LOOK BEYOND “IT GOES UP”

Compare graphs using the same scales



All three panels use x from 0 to 4 and y from 0 to 50. The dots are the five table entries. Each formula also gives outputs between them, so these illustrations show continuous curves on that interval.

What you notice	What it means
The linear graph is straight.	Its slope stays constant: equal horizontal steps give equal vertical changes.
The quadratic graph curves.	Its slope changes; this quadratic has a turning point at (0, 1).
The exponential graph rises faster here.	Its positive output doubles each step. The amount added also gets larger.

Match the domain to the quantity

Use a continuous curve for a continuous quantity. For counts or sequences, include only allowed integer inputs.

LESSON 44 | COMPARE MODELS FOR NOISY DATA

A tiny fitting error is not the whole decision

Return to the fictional signal data from Class 43: times 0, 1, 2, 3, 4, 5 hours and readings 100, 81, 63, 52, 40, 33 units. Three original-output least-squares fits are shown below.

Family and fitted rule	Total squared error
Linear: $L(t) = 95 - 13.4t$	87.20
Quadratic: $Q(t) = 1.5t^2 - 20.9t + 100$	3.20
Exponential: $E(t) \approx 100.291(0.798844)^t$	3.48

Squared error adds $(\text{observed} - \text{predicted})^2$ across the six readings. For this comparison it has squared signal units. The quadratic has the smallest error **on these data**, but the future shape still needs a reason.

Look at the assumptions behind a prediction

$L(10) = -39$

A negative signal is unsuitable for a nonnegative intensity model.

$Q(10) = 41$

The quadratic turns upward after about 6.97 hours.

$E(10) \approx 10.61$

The exponential continues fading toward zero.

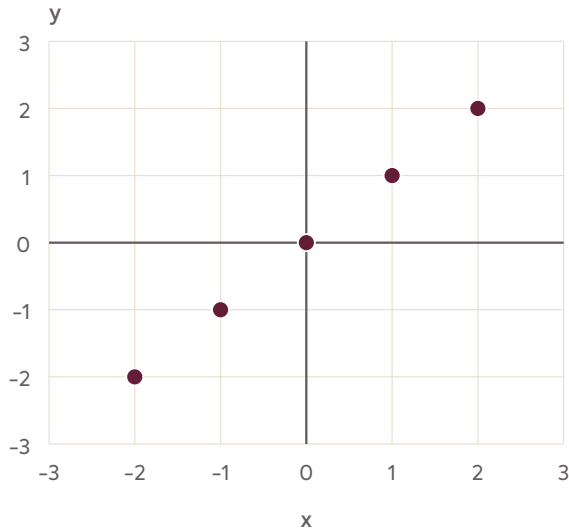
If the process is expected to keep losing roughly the same percentage, the exponential is a reasonable candidate. We need new observations to test that assumption.

For a guided check, refit the table using the linear and quadratic regression forms from earlier lessons. Keep Log Mode off for the exponential comparison. Compare the curves on 0–5 hours before extending them. See the [official regression guide](#).

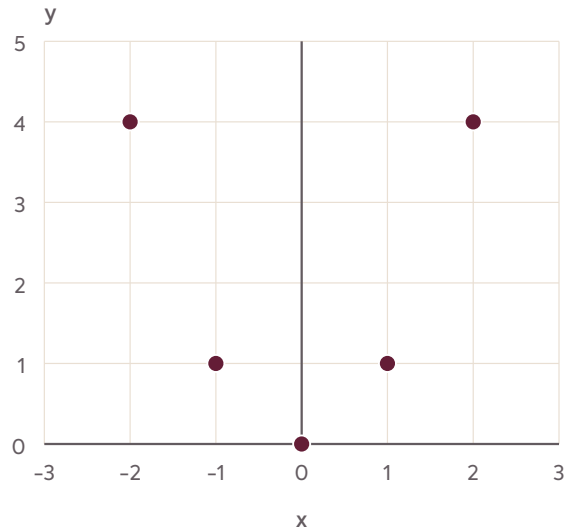
LESSON 44 | WHAT CORRELATION ACTUALLY MEASURES

Association is not a cause

A straight-line association



A perfect curved pattern



Use a graphing calculator's linear-regression analysis on each paired table. For both, $x = -2, -1, 0, 1, 2$. On the left, $y = -2, -1, 0, 1, 2$; on the right, $y = 4, 1, 0, 1, 4$. The linear correlation coefficients are $r = 1$ and $r = 0$, respectively.

r measures linear association

r is between -1 and 1 . Its sign gives the direction of a linear trend; a value near -1 or 1 indicates a strong linear association.

The right-hand dots fit $y = x^2$ perfectly, although $r = 0$. A small r does not rule out a strong **nonlinear** relationship.

Suppose ice-cream sales and pool visits rise together. That association does not prove that buying ice cream causes pool visits. Hot weather could help explain both. Observing a pattern alone does not establish causation.

10-minute quiz . 15-minute explanation . 20-minute guided comparison . 10-minute independent practice . 5-minute exit check. The teacher supplies the quiz separately; the study checks are revision materials.

LESSON 44 | SHORT CHECKS

Your turn

For Questions 1–2, $x = 0, 1, 2, 3$ and the listed patterns are assumed exact within one of the three families.

1. Classify each output list and write a matching rule: (a) 5, 8, 11, 14; (b) 2, 5, 10, 17.

2. Classify 6, 18, 54, 162. Give a rule and predict the output at $x = 4$.

3. For the signal data, which fitted family has the smallest squared error? Why does that alone not settle the best model for continued fading?

4. A scatterplot follows a clear U-shape but has r close to zero. Does that mean there is no association? Explain.

5. A school finds that students who join a club get higher grades. Explain why this does not prove the club caused the higher grades.

6. Why is a signal prediction at $t = 10$ less secure than one at $t = 2.5$ when observations cover only 0–5 hours?

LESSON 44 | WORKED ANSWERS

Choose a model and give a reason

1. (a) Differences are 3, 3, 3: **linear**, $y = 5 + 3x$.

(b) First differences are 3, 5, 7, so second differences are 2, 2: **quadratic**, $y = x^2 + 2x + 2$. Substituting $x = 0, 1, 2, 3$ gives the stated outputs.

2. Ratios are 3, 3, 3: **exponential**, $y = 6 \times 3^x$. At $x = 4$, $y = 6 \times 81 = 486$.

3. The **quadratic** has the smallest error, 3.20 versus about 3.48 for the exponential. But its upward turn would contradict an assumption of continued fading. Context, model behavior and further data also matter.

4. **No**. r measures the strength of a **linear** association. A U-shaped pattern can have strong nonlinear association with r near zero.

5. Students may differ in motivation, prior grades or available support before joining. These or other factors could explain the association. The observation alone does not establish a causal effect.

6. $t = 10$ is **extrapolation** beyond the observed interval; the conditions or pattern may change. $t = 2.5$ is **interpolation** inside the observed interval. Neither is a guaranteed observation.

I can...

- ☐ Use equal-step differences and ratios appropriately.
- ☐ Compare fitted models using both data and context.
- ☐ Explain what correlation and extrapolation cannot prove.

Next class: review the specific skills your work shows you still need. Curriculum focus: A.4A–C, A.8B, A.9E; earlier graph and model skills.

LESSON 45 | MEET THE IDEA

Cumulative review and targeted readiness practice

Tuesday, April 13, 2027 . 6:00–7:00 PM Central

By the end, I can...

Choose evidence-based review priorities, connect different representations and check my reasoning before the final course assessment.

Use your earlier work and teacher feedback to choose **two priorities**. This shared guide is a balanced review; it does not claim to know your personal gaps.

Chapter	Return here for...
1 . Classes 1–6	Expressions, equations, formulas, functions and one-variable inequalities
2 . Classes 7–14	Slope, line equations, domains, transformations and data
3 . Classes 15–20	Two-variable inequalities and systems
4 . Classes 21–26	Exponents, radicals and polynomial operations
5 . Classes 27–32	Factoring and quadratic graph features
6 . Classes 33–38	Four quadratic-solving methods and modeling
7 . Classes 39–43	Sequences and exponential functions
8 . Class 44	Model choice, correlation and prediction limits

Your teacher can select practice from actual assessment evidence. The one-page formula sheet supports revision, but using a formula correctly still requires choosing a method and checking restrictions.

LESSON 45 | LINEAR REASONING

Keep equations balanced and graphs honest

Solve $3(x - 2) = 2x + 5$

$$3x - 6 = 2x + 5$$

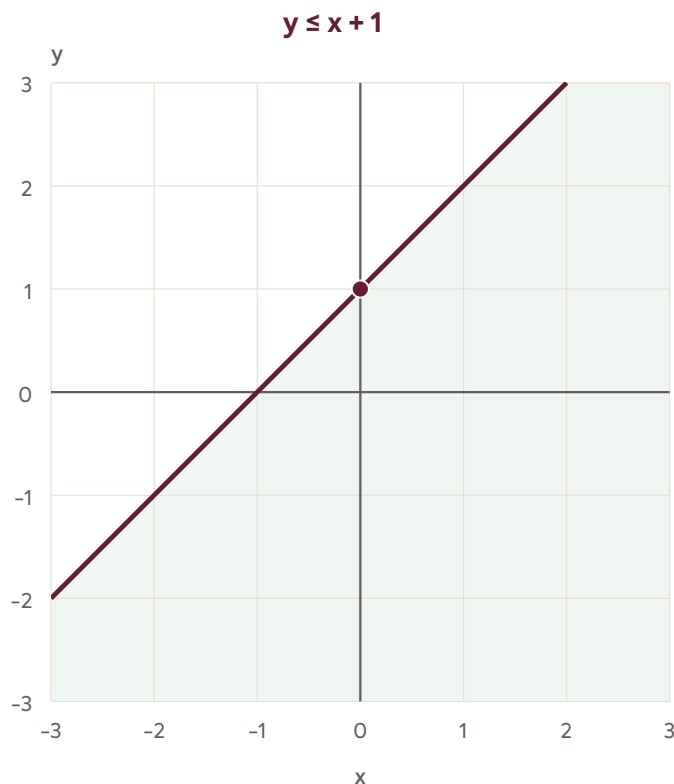
Distribute 3 to both terms.

$$x = 11$$

Subtract $2x$, then add 6 on both sides.

Check: $3(11 - 2) = 27$ and $2(11) + 5 = 27$.

For a formula, isolate the requested variable using the same balance idea. From $A = (1/2)bh$, solve $h = 2A/b$, provided $b \neq 0$. Keep track of any division restriction.



1. Draw the solid line $y = x + 1$.
2. Test $(0, 0)$: $0 \leq 1$ is true.
3. Shade the side containing $(0, 0)$. Boundary points are included.

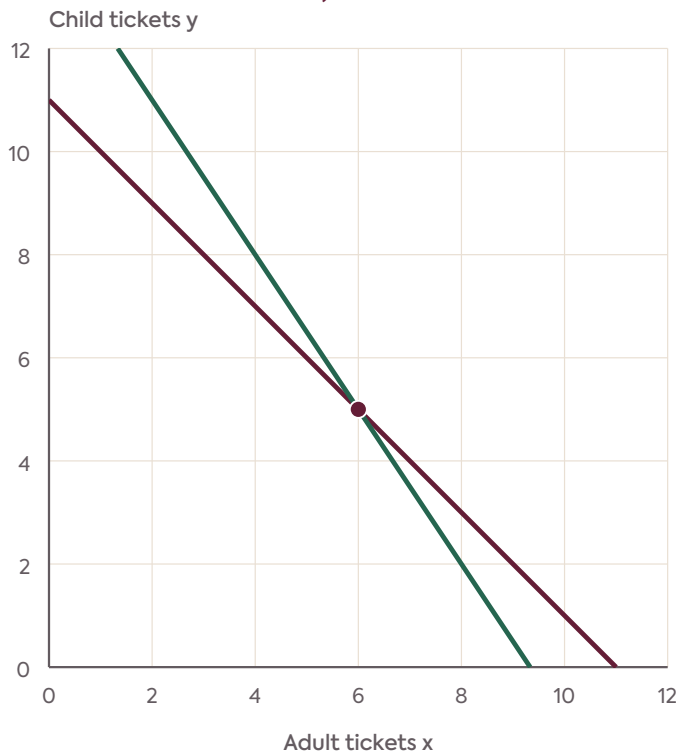
For a strict inequality ($<$ or $>$), draw a dashed boundary. For $\leq$ or $\geq$, use a solid boundary. When multiplying or dividing an inequality by a negative number, reverse its direction.

LESSON 45 | A CONTEXT, TWO EQUATIONS AND A GRAPH

Write the system before solving it

A school event sells 11 tickets: adults pay \$3 and children pay \$2. Total revenue is \$28. Let x be adult tickets and y be child tickets. Both variables must be nonnegative integers.

Two constraints, one intersection



$$x + y = 11$$

Eleven tickets in total.

$$3x + 2y = 28$$

A total price of \$28.

The shared solution is

(6, 5).

Eliminate y

$$2x + 2y = 22$$

Multiply $x + y = 11$ by 2.

$$x = 6; y = 5$$

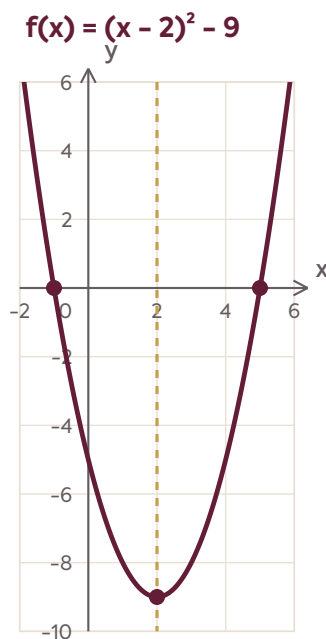
Subtract from $3x + 2y = 28$, then use $x + y = 11$.

Check: $6 + 5 = 11$ and $3(6) + 2(5) = 28$. The continuous lines show the constraints; only whole-number ticket counts are meaningful.

One intersection means one solution. Parallel distinct lines have no solution. Two equations for the same line have infinitely many real solutions before any contextual restrictions are applied.

LESSON 45 | THE SAME FUNCTION IN TWO FORMS

Connect a quadratic graph to its solutions



Vertex: (2, -9)

Axis: $x = 2$

Minimum: -9

Zeros: -1 and 5

Domain: all real x

Range: $y \geq -9$

The dashed gold line shows the axis of symmetry.

Solve $f(x) = 0$ in two ways

$$(x - 2)^2 = 9$$

$$x = 2 \pm 3, \text{ so } x = -1 \text{ or } 5$$

$$x^2 - 4x - 5 = 0$$

$$(x - 5)(x + 1) = 0$$

Use vertex form and isolate the square.

Both signs give a root.

Expand the same function into standard form.

Factoring gives the same two solutions.

The method can change; the solution set should agree

Square roots are efficient when a square is isolated. Factoring is efficient when factors are recognizable. Completing the square and the quadratic formula handle other real-root cases. Check in the original equation.

LESSON 45 | SYMBOLIC AND EXPONENTIAL CHECKS

Watch the structure of each expression

Simplify with exponent laws

$(6x^3)(2x^2)/(3x)$
 $12x^5/(3x) = 4x^4$

Multiply coefficients and add exponents in the numerator.
Divide coefficients and subtract exponents; $x \neq 0$.

Simplify a numerical radical

$\sqrt{72} = \sqrt{36 \times 2}$
 $\sqrt{72} = 6\sqrt{2}$

Find a perfect-square factor.
Keep an exact value until an approximation is needed.

Spot the structure	Use the matching idea
$x^2 + 5x + 6$	Factor to $(x + 2)(x + 3)$: product 6, sum 5.
$P(t) = 300(0.9)^t$	Starts at 300 and decreases by 10% per time unit.
4, 7, 10, ... ; first index 1	Arithmetic: $a_n = 4 + 3(n - 1)$.
4, 8, 16, ... ; first index 1	Geometric: $a_n = 4 \times 2^{n-1}$.

For unrestricted $P(t)$ above, the range is $P > 0$. If the context allows only $t \geq 0$, the range becomes $0 < P \leq 300$. Sequence rules in the table use $n = 1, 2, 3, \dots$.

LESSON 45 | BALANCED MIXED REVIEW

Your turn

These are study questions, not the final assessment. Work on the teacher-selected priorities first. Use graph paper where needed; show the reason for each method.

1. Solve $4(x - 1) = 2x + 10$, then check your solution.

2. Find the line through $(0, 2)$ and $(3, 8)$. Sketch its graph. For $y \geq 2x + 2$, state the boundary style and which side to shade.

3. Solve $x + y = 9$ and $2x + 3y = 23$. Verify both equations.

4. For $f(x) = (x - 1)^2 - 4$, state the vertex, axis of symmetry, zeros and unrestricted range.

5. A positive quantity starts at 200 and loses 10% per hour. Write a model and predict its value after 2 hours.

6. Simplify $\sqrt{(50)}$ and $(2x^3)(3x^2)/x$. State any restriction inherited from the original expression.

7. For 5, 10, 20, ... with first index $n = 1$, write explicit and recursive rules. Find the fifth term.

LESSON 45 | WORKED ANSWERS AND READINESS

Use your answers to choose the next step

1. $4x - 4 = 2x + 10$, so $2x = 14$ and $x = 7$. Check: $4(6) = 24$ and $2(7) + 10 = 24$.
2. Slope = $(8 - 2)/(3 - 0) = 2$. The line is $y = 2x + 2$. Plot $(0, 2)$ and $(3, 8)$, then draw the line through them. For $y \geq 2x + 2$, use a **solid boundary and shade above**. The origin fails $0 \geq 2$, so it is not shaded.
3. Double the first equation: $2x + 2y = 18$. Subtract to get $y = 5$, then $x = 4$. Checks: $4 + 5 = 9$ and $8 + 15 = 23$.
4. Vertex $(1, -4)$; axis $x = 1$. Solve $(x - 1)^2 = 4$ to obtain zeros **-1 and 3**. The graph opens upward, so the range is $y \geq -4$.
5. $P(t) = 200(0.9)^t$, where t is hours since the start. $P(2) = 200 \times 0.81 = 162$.
6. $\sqrt{(50)} = 5\sqrt{(2)}$. The second expression is $6x^5/x = 6x^4$, with $x \neq 0$ inherited from the original denominator.
7. Explicit: $a_n = 5 \times 2^{n-1}$, $n = 1, 2, \dots$. Recursive: $a_1 = 5$; $a_n = 2a_{n-1}$, $n = 2, 3, \dots$. $a_5 = 80$.

Before the final

- ☐ Choose two priorities using real errors or teacher feedback.
- ☐ Practice explaining one graph and checking one equation.
- ☐ Confirm the assessment link, scratch paper and permitted tools.

Next class: the internal course assessment, followed by feedback and a short personal practice plan. A single score samples learning; it does not replace the evidence collected across the course.

LESSON 46 | BEFORE THE ASSESSMENT

Final cumulative assessment and next learning steps

Thursday, April 15, 2027 . 6:00–7:00 PM Central

This is the Cambridge Veritas **internal course assessment**: 20 original items, worth 24 points in total. It samples the Algebra I work you have studied. Follow your teacher’s instructions and complete your own work.

Time	What happens
5 minutes	Instructions, tool check and setup
40 minutes	Independent assessment
10 minutes	Reflection and initial debrief
5 minutes	Next steps

Confirm permitted tools before starting

Use only the calculator, reference material and other resources your teacher authorizes for this internal assessment. Ask before starting if any instruction is unclear.

Teacher-approved calculator or tool:

Permitted reference sheet / other agreed arrangements:

Keep scratch paper ready. Show the work requested and read the units and rounding instructions. If a technical problem occurs, tell your teacher immediately. Submit through the assessment route provided by your teacher.

Release note: this page is the pre-assessment handout. Pages 2–4 are for use after teacher review. The assessment questions and answer key are separate controlled materials.

LESSON 46 | AFTER TEACHER REVIEW

Read the evidence, then choose a priority

Complete this page with your teacher once your work has been reviewed. Written explanations may need marking after the live class. These spaces intentionally contain no student results.

Student name and review date:

Final course assessment: _____ / 24 points

What the result means

This is a score on our internal assessment. It is not a school course grade, a STAAR scale score or a prediction of passing a state exam. Use it alongside your earlier classwork, homework and teacher feedback.

A strength I can show with a specific piece of work:

A skill to revisit and the item or earlier work that shows it:

My teacher's explanation of the next step:

An unanswered item, a sign slip and a misunderstood method may need different responses. Name the actual issue before choosing more practice.

LESSON 46 | AFTER TEACHER REVIEW

Correct the reasoning, not just the answer

This illustration is a revision example, not a question from your final assessment. Use the same routine when your teacher returns a selected correction.

Illustration: solve $-3(x - 2) = 12$

$$x - 2 = -4$$

Divide both sides by -3 .

$$x = -2$$

Add 2 to both sides.

$$-3(-2 - 2) = 12$$

Check: $-3 \times (-4) = 12$.

A common error is dividing 12 by -3 and writing $+4$. Explain the sign rule, then verify the corrected solution in the original equation.

My assigned correction: item / skill / chapter

The first step I need to change, and why:

My corrected working:

How I checked the result:

Use the question your teacher returns to you. A copied answer alone does not show whether the underlying skill is secure.

LESSON 46 | AFTER TEACHER REVIEW

Leave with a small, useful practice plan

Choose one or two priorities from real feedback. Short, focused follow-up can be more useful than repeating an entire chapter. This worksheet does not add a required live class.

Priority 1: skill and evidence

Priority 2, if needed: skill and evidence

Next action	What to do
Revisit	Open the named lesson and explain one worked example in your own words.
Practice	Complete a short teacher-selected set on that skill.
Check independently	Try a different example with the answer hidden; show your method.
Review progress	Ask your teacher to check the reasoning and decide what comes next.

Assigned lesson / practice and agreed check-in date:

One mathematical idea I can now explain clearly:

Keep using the habits you built

Name the quantities. Choose a representation. Show your steps. Check the result. Explain why it makes sense. The next learning step should follow your actual work.

USE EVIDENCE, NOT A GUESS

Chapter 8 quick reference

Decision	A reliable check
Exact table	Use equally spaced inputs. Constant first differences suggest a line; constant nonzero second differences suggest a quadratic.
Exponential pattern	Check a constant positive ratio between nonzero consecutive outputs. State the initial input and time unit.
Noisy data	Do not demand exact differences or ratios. Compare fitted curves, errors, context and behavior outside the observed interval.
Correlation	r measures linear association. r near zero does not rule out a nonlinear pattern; association alone is not causation.
A graph	Label axes, scales and units. Respect continuous or discrete inputs; use solid or dashed inequality boundaries correctly.
An equation	Preserve equality at every step. Verify each candidate in the original equation and apply context restrictions.
A system	The solution must satisfy both equations or all stated inequalities, not just one condition.
A quadratic	Connect forms: factors reveal zeros; vertex form reveals the turning point. Choose a solving method suited to the structure.
Personal review	Choose priorities from actual marked work. Correct the first faulty step and attempt a different example independently.

Explain the choice of method, show enough working to follow it, and finish with a check. These habits connect all eight chapters.

COURSE COMPLETION

Study guidance and references

Original course materials

These are Cambridge Veritas supplementary study notes aligned to the taught Texas Algebra I content. They are not an official TEA publication. The sequence and dates follow our agreed course calendar.

Assessment and personal feedback

Class 44 has a separate teacher-provided quiz. Class 46 uses the agreed internal 20-item, 24-point assessment plan. The student PDFs contain neither that test nor its unpublished answers. The Class 46 worksheets are completed after review; they do not contain invented student results.

Graphs, data and model fitting

Graphs are calculated from the displayed equations. All datasets and numerical stories are fictional teaching examples. The comparison reuses the Class 43 signal data and original-output least-squares fits. Squared errors are compared on the same observations and output scale.

The separate formula sheet is a course revision aid, not an official STAAR reference sheet. Your teacher sets permitted tools for internal assessments; schools provide the instructions for state testing.

Curriculum alignment

Class 44 consolidates A.4A–C, A.8B and A.9E with earlier graph and modeling work. Class 45 reviews selected skills from Chapters 1–7; Class 46 samples learning and supports correction. The relevant mathematical processes A.1A–G run throughout. The corresponding advanced Grade 8 expectations are in §111.31(d).

Official references

Texas Education Agency: [Standard Algebra I mathematics TEKS, §111.39](#).

Texas Education Agency: [Advanced Grade 8 Algebra I, §111.31, adopted 2025](#).

Texas Education Agency: [STAAR mathematics resources and official reference materials](#).

Desmos: [Official regression instructions](#).

Curriculum/resources checked September 10, 2026. Chapter 8 edition: 1.0.